上节课回顾

经典理论

$$\frac{d\vec{v}}{dt}=-\frac{1}{\rho}\nabla p+\vec{g}-2\vec{\Omega}\times\vec{v}+\vec{F}$$

$$\frac{\partial\rho}{\partial t}=-\nabla\cdot(\rho\vec{V})$$

$$P\alpha=RT$$

$$Q=Cp\frac{dT}{dt}-\alpha\frac{dp}{dt}$$

$$\frac{\partial\rho q}{\partial t}=-\nabla\cdot(\rho\vec{V}q)+\rho(E-C)$$

\+

最新研究成果

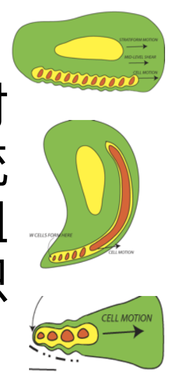
\+

实际应用

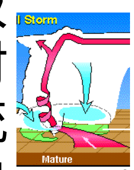
中尺度分析基础

→ 对流触发

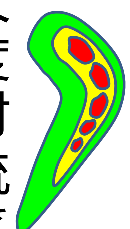
→ 对流组织



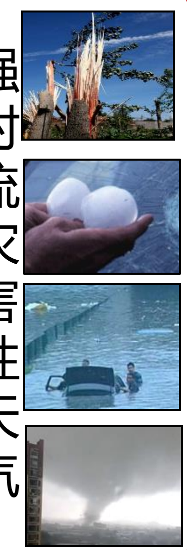
→ 超级对流单体



→ 中尺度对流系统



→ 强对流灾害性天气



→ 强对流天气的数值模拟

初始场
↓
数值模式
↓
预报场

课程材料

1. 请查看课程网站，上完每一章后将上传相应材料（安排、课件、作业）

https://qiuyang50.github.io/_pages/mesoscale_2024fall/

中尺度动力学

2. 请加入课程微信群（通知、交流），群内请注明真实姓名

群聊: 中尺度动力学

第一章 中尺度分析基础

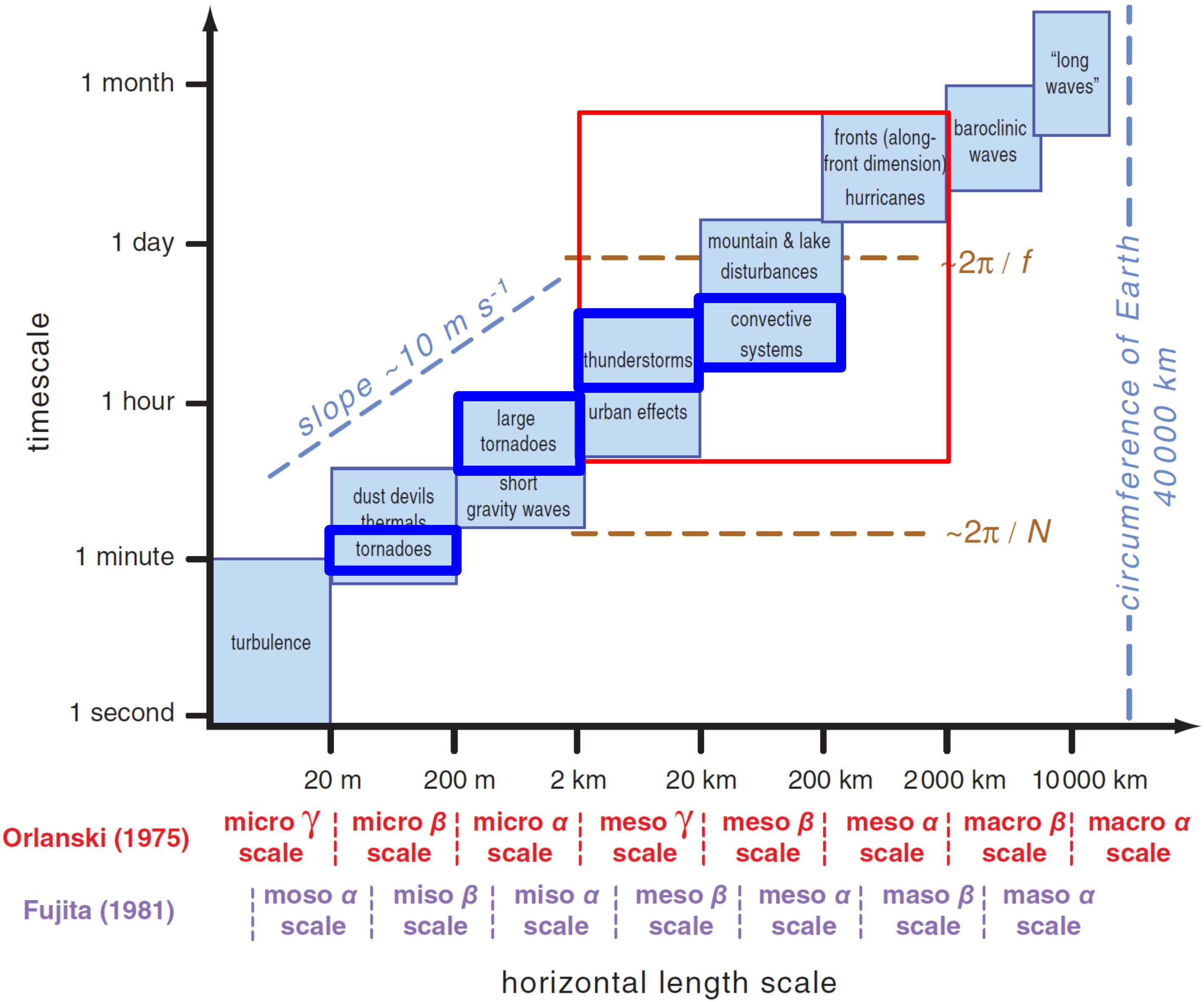
主要内容

天气和天气系统

天气：某一个地区距离地表较近的大气层在短时间内的具体状态。大气中气象要素的空间分布可表现为各种瞬息万变的天气现象。

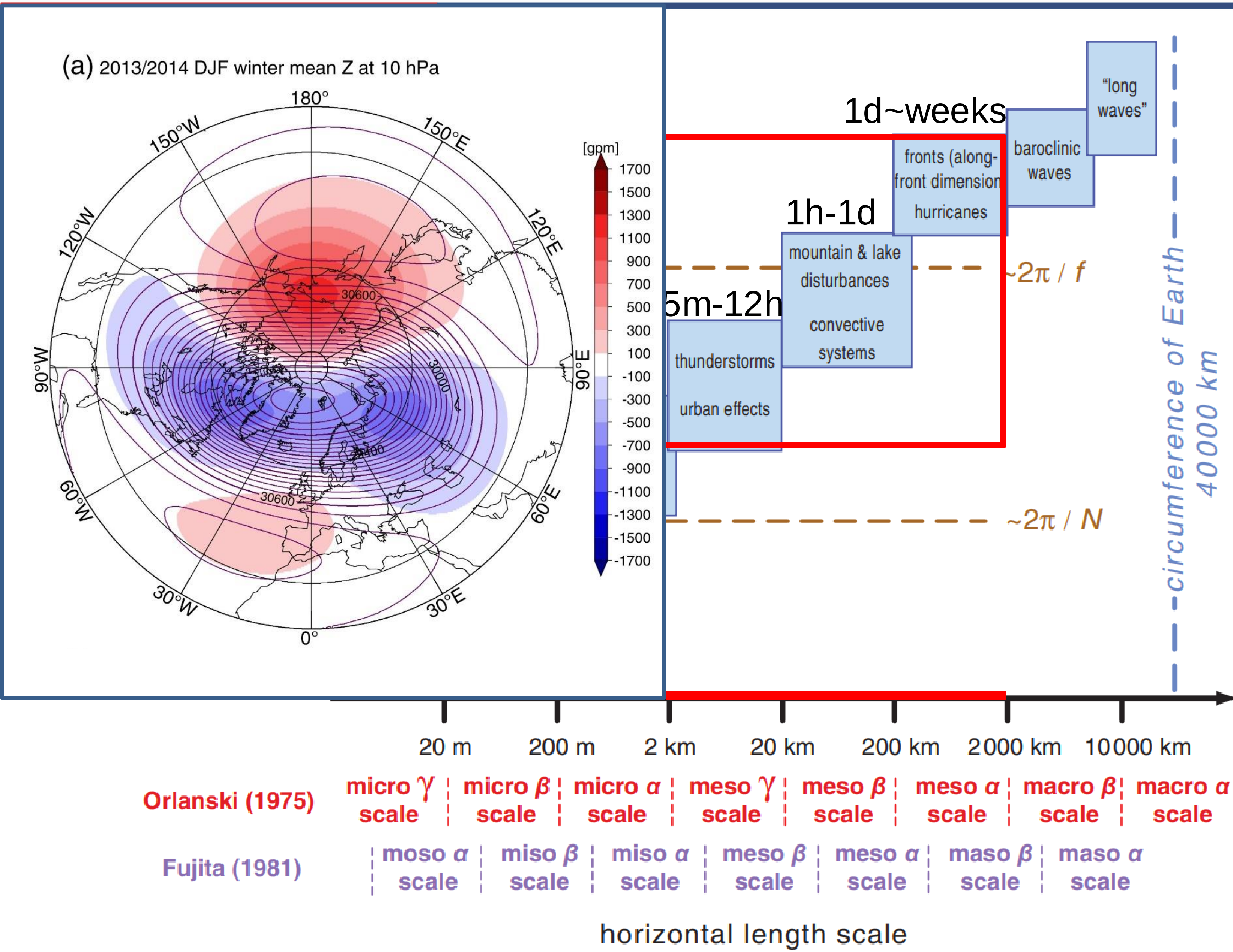
天气系统：引起各种天气变化和分布的高压、低压和高压脊、低压槽等具有典型特征的大气运动系统。

什么是中尺度?

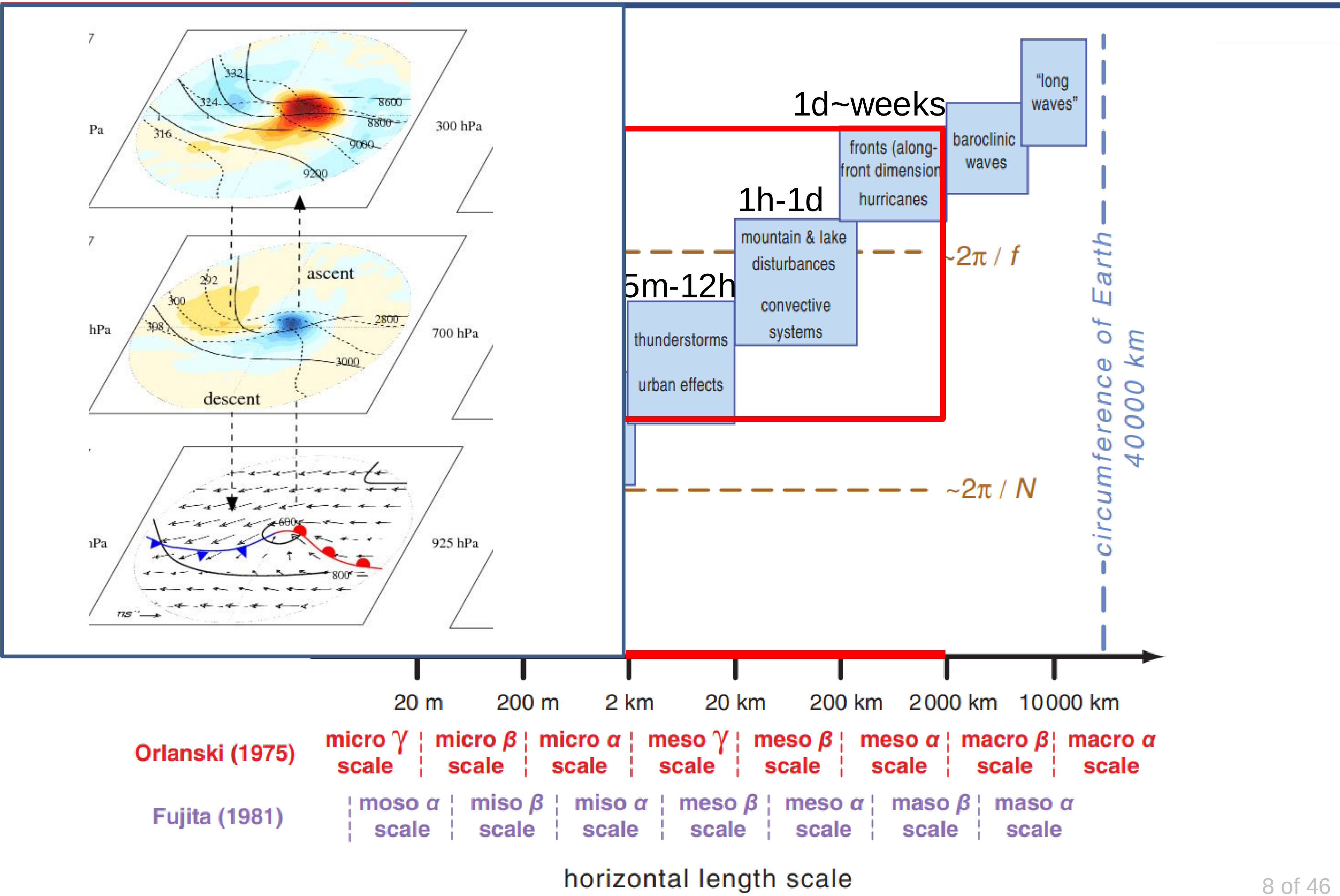


(MR2010)

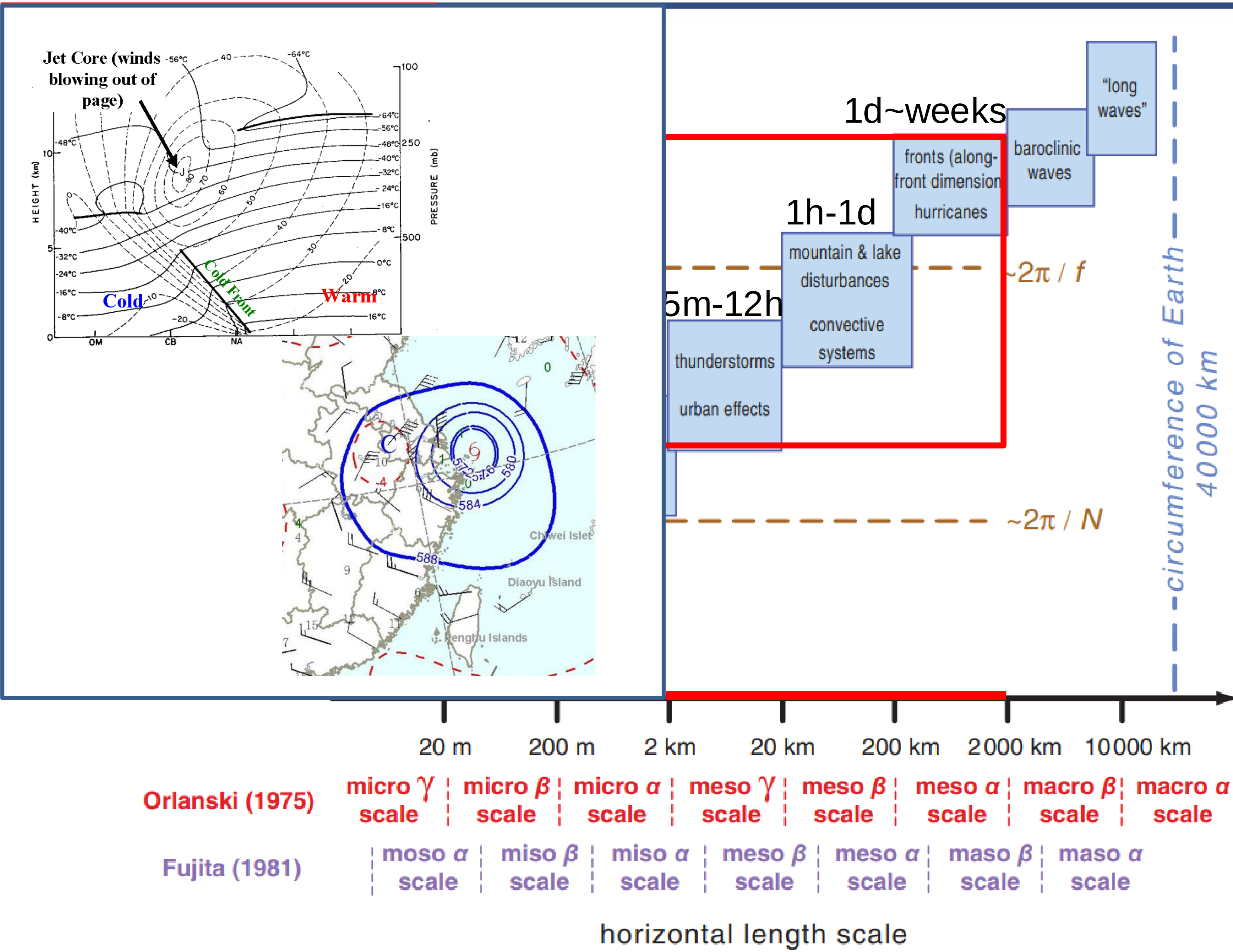
尺度分类: 大尺度驻波


(a) 2013/2014 DJF winter mean Z at 10 hPa
180°
150°W
150°E
120°W
120°E
90°W
90°E
60°W
60°E
30°W
30°E
0°
30600
30000
[gpm]
1700
1500
1300
1100
900
700
500
300
100
-100
-300
-500
-700
-900
-1100
-1300
-1500
-1700
"long waves"
1d~weeks
baroclinic waves
fronts (along-front dimension
hurricanes
1h-1d
mountain & lake disturbances
convective systems
~2π / f
5m-12h
thunderstorms
urban effects
~2π / N
circumference of Earth
40000 km
20 m
200 m
2 km
20 km
200 km
2000 km
10000 km
Orlanski (1975)
micro γ scale
micro β scale
micro α scale
meso γ scale
meso β scale
meso α scale
macro β scale
macro α scale
Fujita (1981)
moso α scale
miso β scale
miso α scale
meso β scale
meso α scale
maso β scale
maso α scale
horizontal length scale

尺度分类：中尺度锋面


300 hPa
700 hPa
925 hPa
ascent
descent
1d~weeks
1h-1d
5m-12h
"long waves"
baroclinic waves
fronts (along-front dimension
hurricanes
mountain & lake disturbances
convective systems
thunderstorms
urban effects
~2π / f
~2π / N
circumference of Earth
40000 km
20 m
200 m
2 km
20 km
200 km
2000 km
10000 km
Orlanski (1975)
micro γ scale
micro β scale
micro α scale
meso γ scale
meso β scale
meso α scale
macro β scale
macro α scale
Fujita (1981)
moso α scale
miso β scale
miso α scale
meso β scale
meso α scale
maso β scale
maso α scale
horizontal length scale

尺度分类：台风



Jet Core (winds blowing out of page)
Cold
Cold Front
Warm
HEIGHT (km)
PRESSURE (mb)
Chiwei Islet
Diaoyu Island
Penghu Islands
"long waves"
baroclinic waves
1d~weeks
fronts (along-front dimension)
hurricanes
1h-1d
mountain & lake disturbances
convective systems
5m-12h
thunderstorms
urban effects
~2π / f
~2π / N
circumference of Earth
40000 km
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macro α scale
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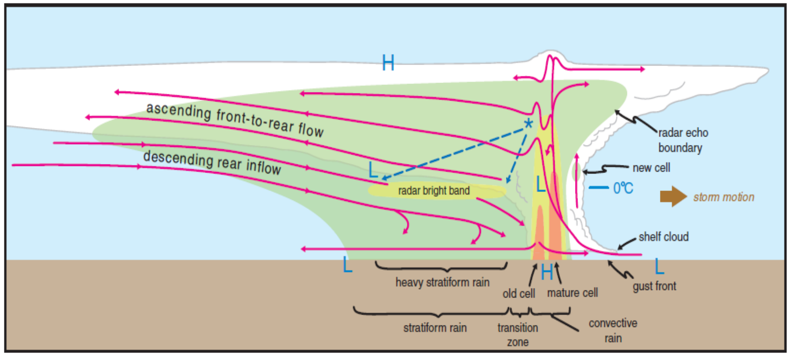
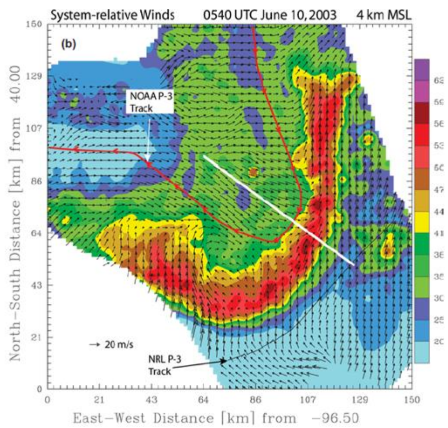
moso α scale
miso β scale
miso α scale
meso β scale
meso α scale
maso β scale
maso α scale
horizontal length scale

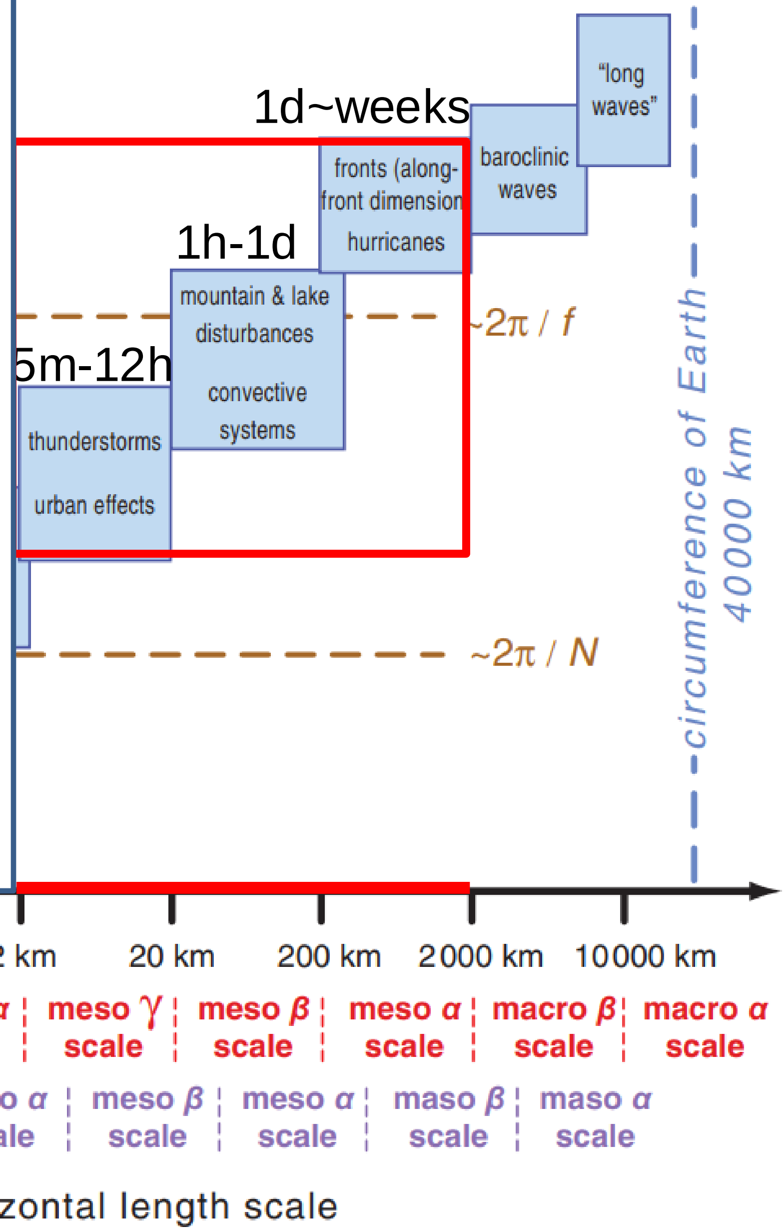
尺度分类：飑线



H
ascending front-to-rear flow
descending rear inflow
radar echo boundary
new cell
0℃
storm motion
radar bright band
shelf cloud
L
H
heavy stratiform rain
old cell
mature cell
gust front
stratiform rain
transition zone
convective rain

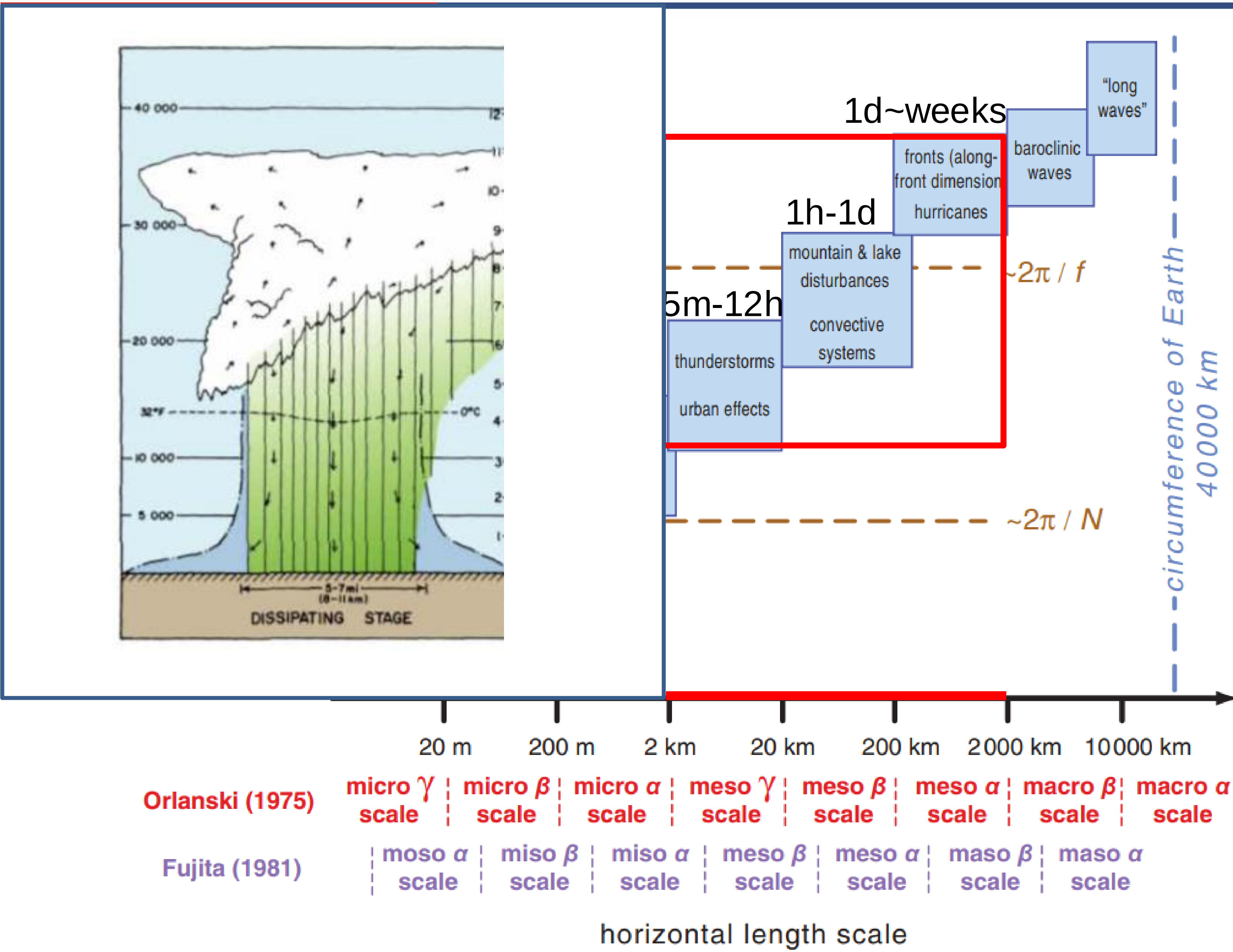


System-relative Winds 0540 UTC June 10,2003 4 km MSL
(b)
NOAA P-3 Track
NRL P-3 Track
20 m/s
North–South Distance [km] from 40.00
East–West Distance [km] from −96.50



1d~weeks
1h-1d
5m-12h
"long waves"
baroclinic waves
fronts (along-front dimension)
hurricanes
mountain & lake disturbances
convective systems
thunderstorms
urban effects
~2π / f
~2π / N
circumference of Earth 40000 km
20 m
200 m
2 km
20 km
200 km
2000 km
10000 km
Orlanski (1975)
micro γ scale
micro β scale
micro α scale
meso γ scale
meso β scale
meso α scale
macro β scale
macro α scale
Fujita (1981)
moso α scale
miso β scale
miso α scale
meso β scale
meso α scale
maso β scale
maso α scale
horizontal length scale

尺度分类：雷暴


40 000
30 000
20 000
10 000
5 000
32°F
0°C
5-7 mi
(8-11 km)
DISSIPATING STAGE
1d~weeks
1h-1d
5m-12h
thunderstorms
urban effects
mountain & lake disturbances
convective systems
fronts (along-front dimension
hurricanes
baroclinic waves
"long waves"
~2π / f
~2π / N
circumference of Earth
40000 km
20 m
200 m
2 km
20 km
200 km
2000 km
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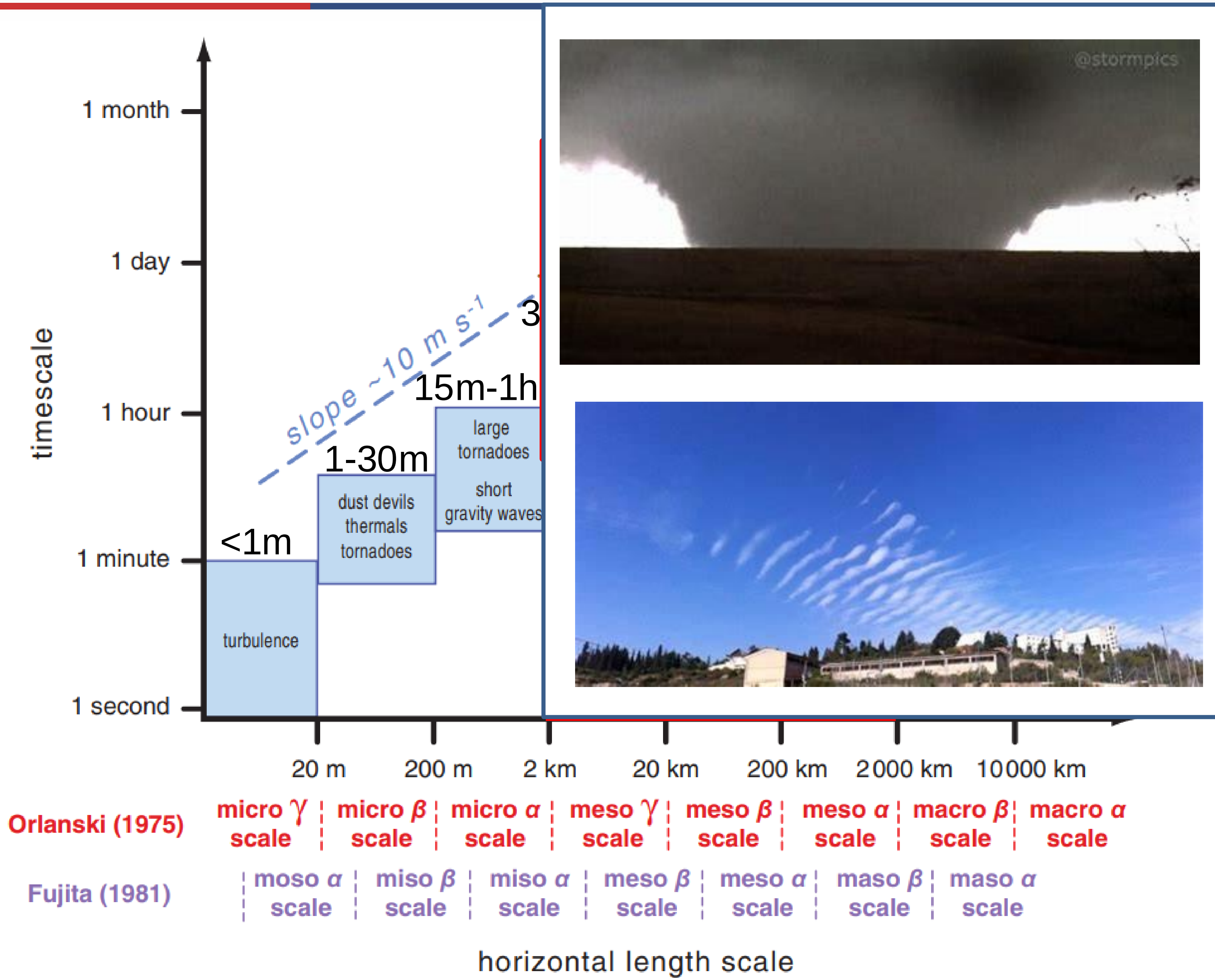
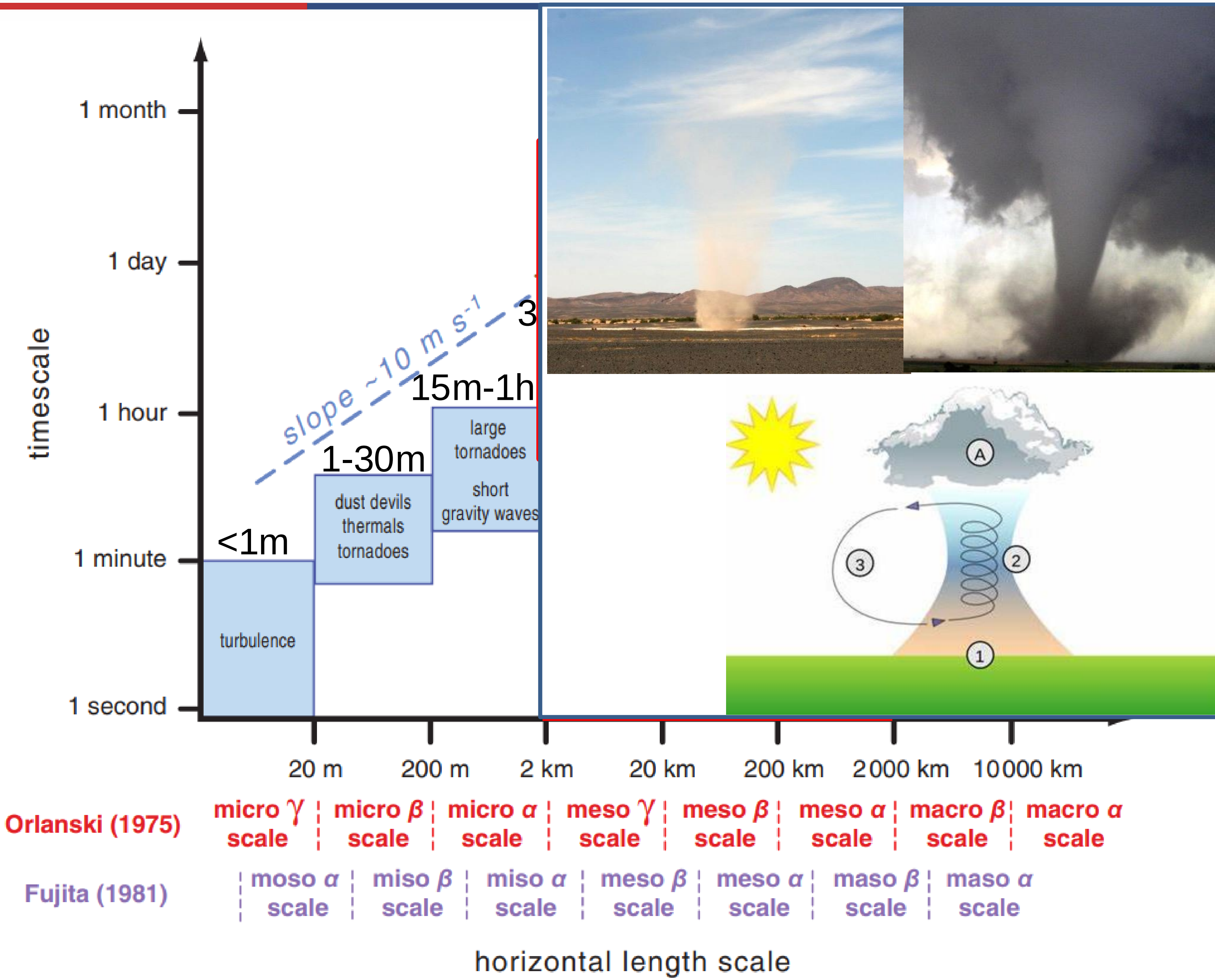
meso β scale
meso α scale
macro β scale
macro α scale
Fujita (1981)
moso α scale
miso β scale
miso α scale
meso β scale
meso α scale
maso β scale
maso α scale
horizontal length scale

尺度分类: 大龙卷、 重力波



1 month
1 day
1 hour
1 minute
1 second
timescale
slope ~10 m s-1
<1m
1-30m
15m-1h
3
turbulence
dust devils
thermals
tornadoes
large
tornadoes
short
gravity waves
20 m
200 m
2 km
20 km
200 km
2000 km
10000 km
Orlanski (1975)
micro γ scale
micro β scale
micro α scale
meso γ scale
meso β scale
meso α scale
macro β scale
macro α scale
Fujita (1981)
moso α scale
miso β scale
miso α scale
meso β scale
meso α scale
maso β scale
maso α scale
horizontal length scale
@stormpics

尺度分类：龙卷


timescale
1 month
1 day
1 hour
1 minute
1 second
slope ~10 m s^{-1}
<1m
1-30m
15m-1h
3
turbulence
dust devils
thermals
tornadoes
large
tornadoes
short
gravity waves
A
1
2
3
20 m
200 m
2 km
20 km
200 km
2000 km
10000 km
Orlanski (1975)
micro γ scale
micro β scale
micro α scale
meso γ scale
meso β scale
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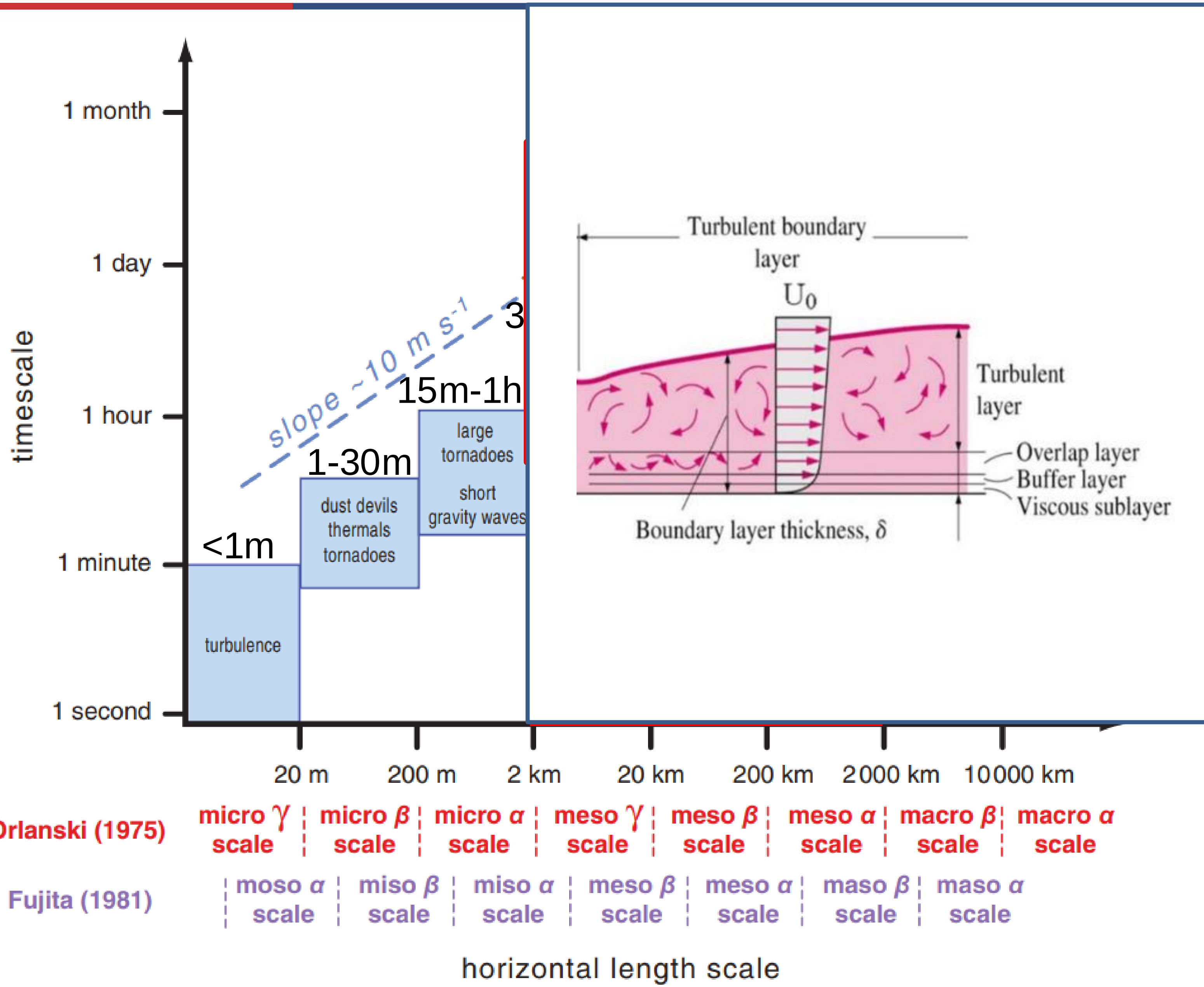
macro β scale
macro α scale
Fujita (1981)
moso α scale
miso β scale
miso α scale
meso β scale
meso α scale
maso β scale
maso α scale
horizontal length scale

尺度分类：湍流


timescale
1 month
1 day
1 hour
1 minute
1 second
slope ~10 m s-1
<1m
turbulence
1-30m
dust devils
thermals
tornadoes
15m-1h
large tornadoes
short gravity waves
3
Turbulent boundary layer
U0
Turbulent layer
Overlap layer
Buffer layer
Viscous sublayer
Boundary layer thickness, δ
20 m
200 m
2 km
20 km
200 km
2000 km
10000 km
Orlanski (1975)
micro γ scale
micro β scale
micro α scale
meso γ scale
meso β scale
meso α scale
macro β scale
macro α scale
Fujita (1981)
moso α scale
miso β scale
miso α scale
meso β scale
meso α scale
maso β scale
maso α scale
horizontal length scale

按尺度划分

小尺度：水平范围2km以下, 生命期为几分钟到几小时.

updraft scale $2\pi/N$

中尺度：水平范围2~2000km, 生命期为几小时到几天。

大尺度：水平范围2000km以上, 生命期为几天到十几天。

$\mathrm{N}H/f$ $2\pi/f$

天气尺度：200~2000km 生命期为一天到几天 (台风、锋面、气旋、反气旋等),

人们有时把等于或大于天气尺度的天气系统统称为大尺度天气系统.

中尺度的特殊性

a. 基本方程组最接近于原始方程

大尺度: $\frac{dw}{dt}$, $\vec{v_a}\cdot\nabla\alpha$ 可以忽略

小尺度: 科氏力，甚至水平气压梯度力可以忽略

$$
\begin{cases}
\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv \\
\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu \\
\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + g \\
\frac{d\rho}{dt} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0 \\
p = \rho RT \\
\frac{d\theta}{dt} = -\frac{\theta}{C_p T}\frac{dQ}{dt}
\end{cases}
$$

b. 运动受多种不稳定性控制

大尺度: 斜压不稳定

中尺度: 静力不稳定，对称不稳定，正压不稳定（Kelvin-Helmholtz不稳定）

小尺度: 静力不稳定

c. 时间尺度

大尺度: 惯性震荡 ($\frac{2\pi}{f}$ ~17 h)

小尺度: 纯浮力震荡 ($\frac{2\pi}{N}$~ 10 min)

对称不稳定

惯性不稳定的判据（水平判据）：

$$M = u_g - fy$$ 绝对动量(此处定义和第5章差负号)

$$\frac{\partial M}{\partial y} > 0$$ 不稳定

$$\frac{\partial M}{\partial y} < 0$$ 稳定

干对流不稳定的判据（垂直判据）：

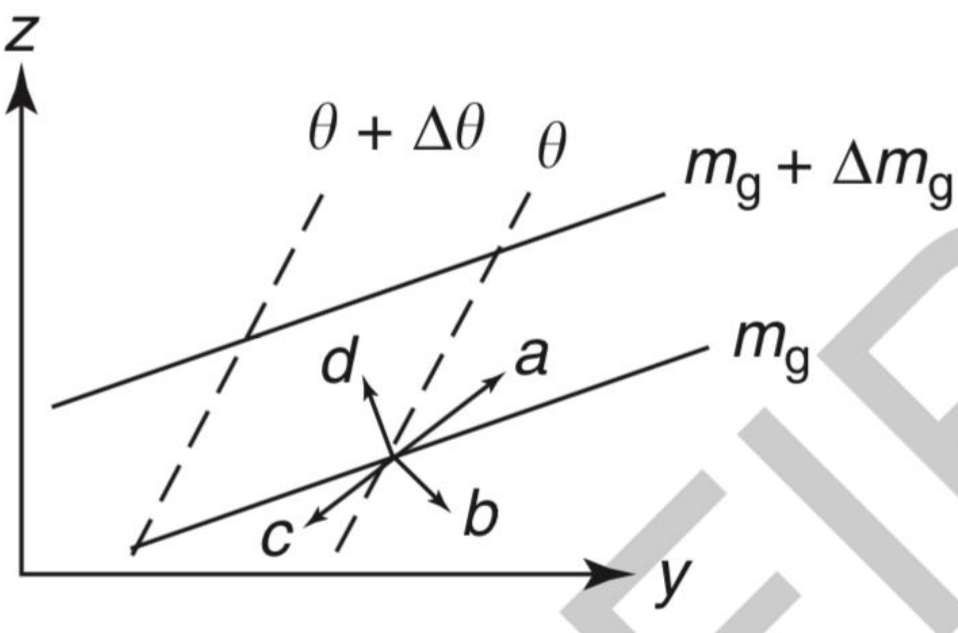
$$N^2 = \frac{g}{\theta_0}\frac{\partial \theta_0}{\partial z} = \frac{g}{T_0}(\Gamma_d - \Gamma)$$

现考虑在y-z平面上的运动，如果运动可以沿着倾斜的方向，则稳定度判据有何不同?

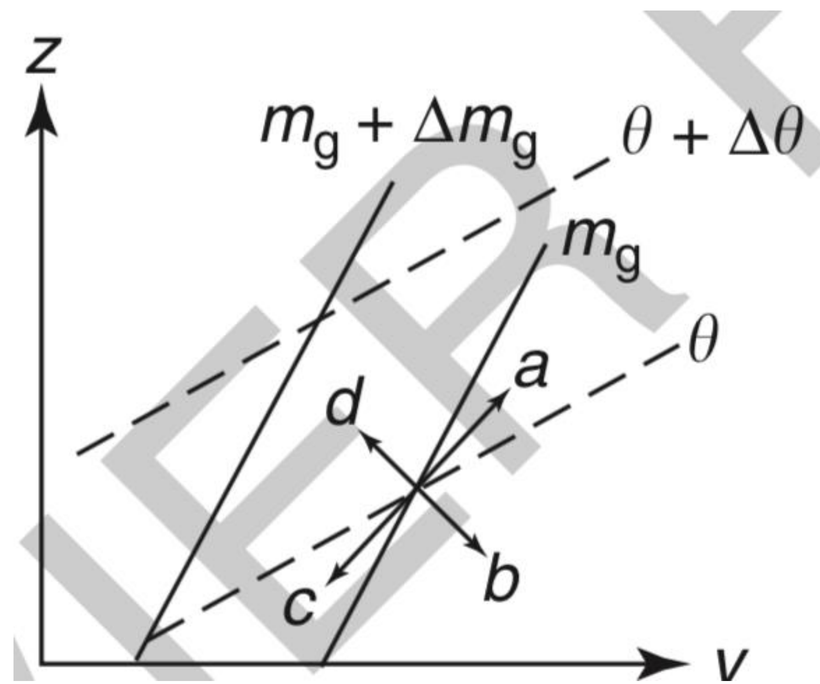
θ等值线和M_g等值线夹角为正：对称不稳定

沿着a, c方向，
对流不稳定且惯性不稳定

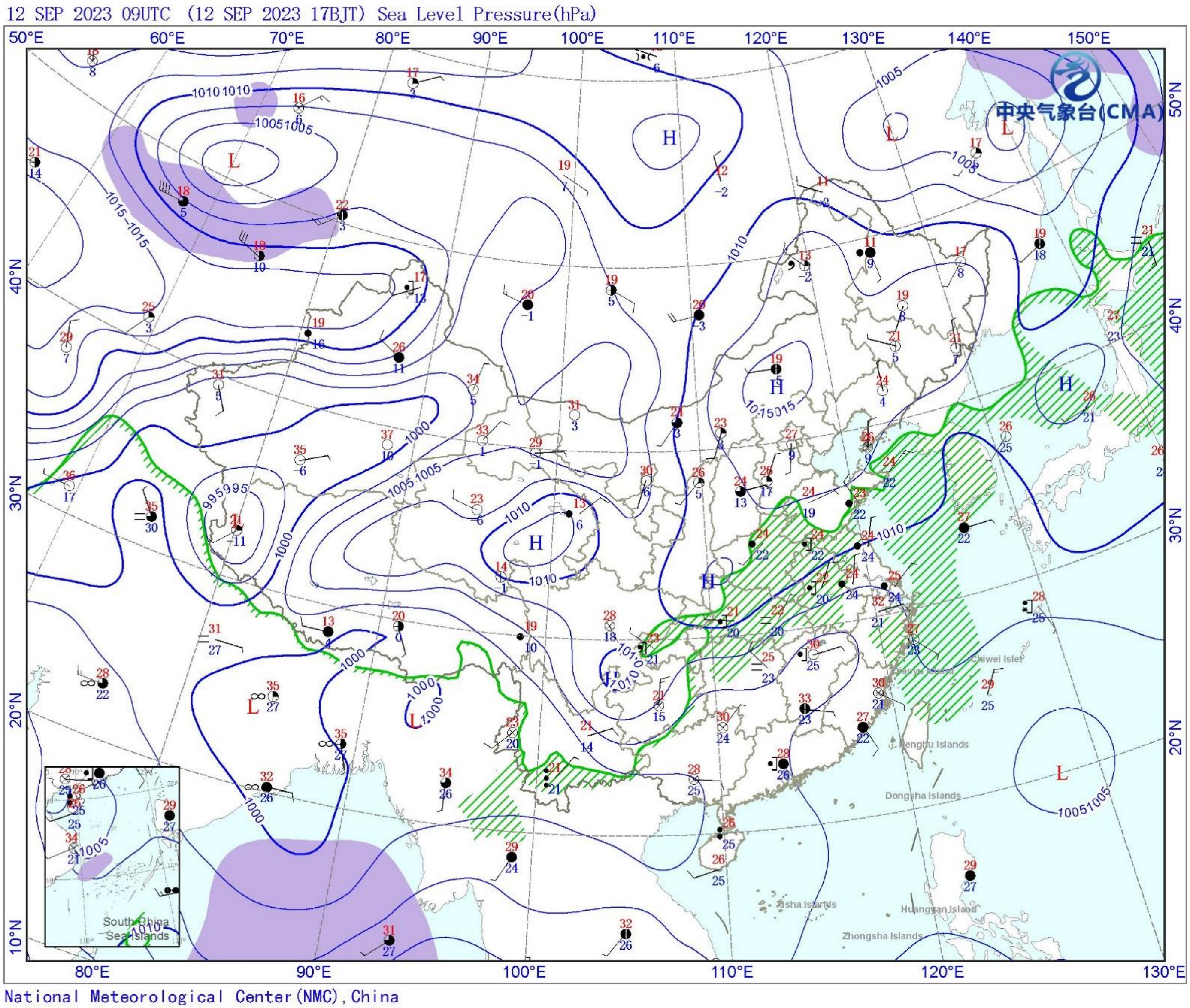
$$\frac{\delta\theta}{\delta z}|_s < 0 \qquad \frac{\delta m_g}{\delta y}|_s > 0$$



θ等值线和M_g等值线夹角为负：对称稳定



大尺度的气压变化



12 SEP 2023 09UTC (12 SEP 2023 17BJT) Sea Level Pressure(hPa)
中央气象台(CMA)
South China Sea Islands
National Meteorological Center(NMC), China

中尺度的气压变化

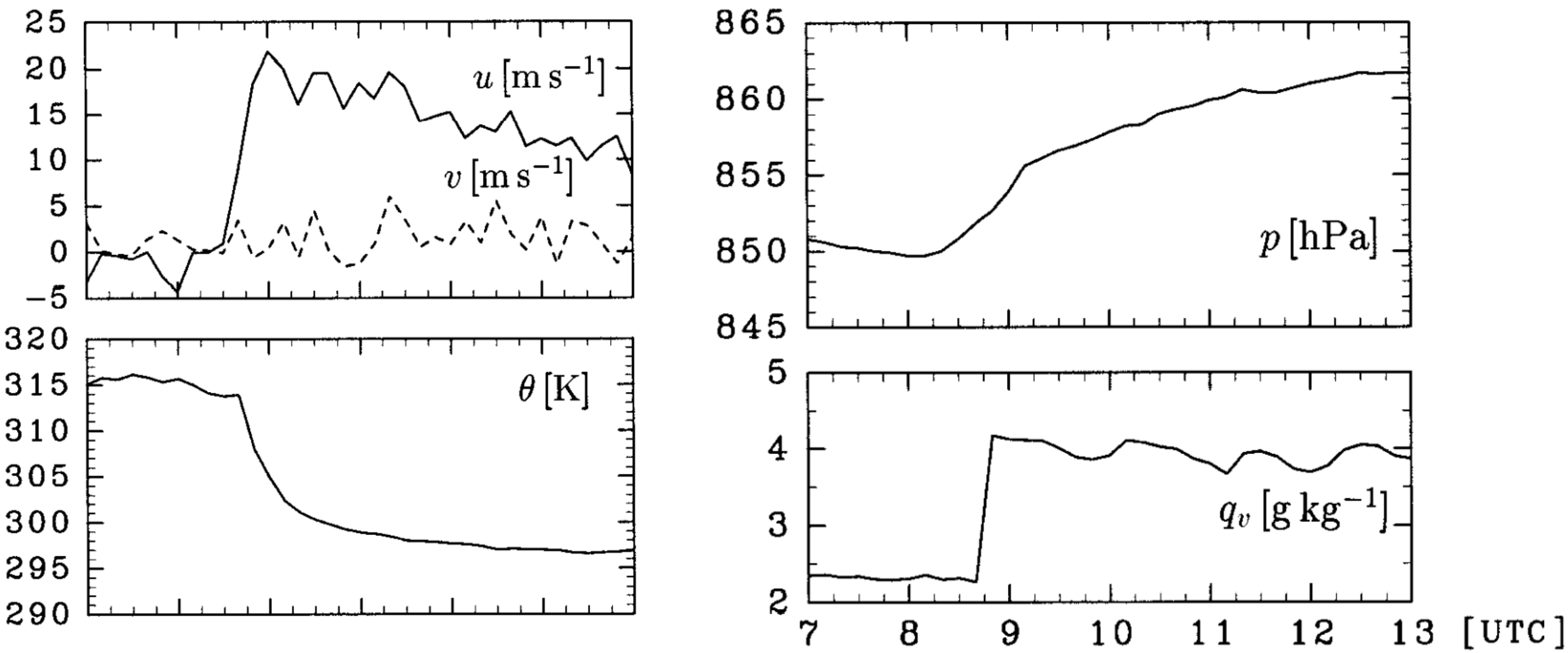


FIG. 6. Time series of surface variables observed at Minqin AWS during the squall line passage. Wind components u and v indicate parallel and normal to the squall line motion (from 304°), respectively. The arrival of the gust front is at about 0840 UTC.

（TAKEMI，1999，MWR）

中尺度和大尺度的动力学差别

a. 水平方向

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv + F_u \quad \text{(E1.1)}$$

	大尺度	中尺度
O (fv)	$10^{-4}\ \mathrm{s^{-1}}\ .\ 10\ \mathrm{m\,s^{-1}}$ $\sim 10^{-3}\ \mathrm{m\,s^{-2}}$	$\sim 10^{-3}\ m\,s^{-2}$
O $(-\frac{1}{\rho}\frac{\partial p}{\partial x})$	$\frac{1}{1\ \mathrm{kg.m^{-3}}} \cdot \frac{10\ \mathrm{mb}}{1000\ \mathrm{km}}$ $= \frac{1}{1\ \mathrm{kg.m^{-3}}} \cdot \frac{10^3\mathrm{kg{\cdot}m}}{\mathrm{s^{-2}m^2 10^6 m}}$ $= 10^{-3}\ \mathrm{m \cdot s^{-2}}$	$\frac{1}{1\ \mathrm{kg.m^{-3}}} \cdot \frac{10\ \mathrm{mb}}{10\ \mathrm{km}}$ $= \frac{1}{1\ \mathrm{kg.m^{-3}}} \cdot \frac{10^3\mathrm{kg{\cdot}m}}{\mathrm{s^{-2}m^2 10^4 m}}$ $= 10^{-1}\ \mathrm{m \cdot s^{-2}}$

中尺度和大尺度的动力学差别

大尺度

$$\Rightarrow \frac{du}{dt}$$ 很小

$$v = v_a + v_g$$

$$f v_a = f v - f v_g = f v - \frac{1}{\rho}\frac{\partial p}{\partial x}$$

$\Rightarrow$ 非地转风很小

为准地转平衡

$$R_0 = \frac{U}{fL} \quad \frac{10\ \mathrm{m}\cdot\mathrm{s}^{-1}}{10^{-4}\mathrm{s}^{-1}\cdot 10^{6}\ \mathrm{m}}$$

$$= 10^{-1} \ll 1$$

中尺度

加速度和非地转风都很大

地转、准地转、梯度风平衡近似都不适用

$$\frac{10\ \mathrm{m}\cdot\mathrm{s}^{-1}}{10^{-4}\mathrm{s}^{-1}\cdot 10^{4}\ \mathrm{m}}$$

$$= 10 \geq 1$$

中尺度和大尺度的动力学差别

b. 垂直方向

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + 2\Omega u\cos\phi - g + F_w \quad \text{(E1.2)}$$

$$= -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

如果 $\frac{dw}{dt}$ 可以忽略，$-\frac{1}{\rho}\frac{\partial p}{\partial z} = g$ 为静力平衡

- 是不是所有的中尺度天气系统都不满足静力平衡?
- 天气尺度呢?

静力平衡需要满足什么条件?

$\frac{dw}{dt}$ 与 $-\frac{1}{\rho}\frac{\partial p}{\partial z}$ 尺度比较

中尺度和大尺度的动力学差别

$\frac{dw}{dt}$ 与 $-\frac{1}{\rho}\frac{\partial p}{\partial z}$ 尺度比较

参考大气：定常，水平均匀，满足静力平衡

$$p = \bar{p}(z) + p'(x, y, z, t)$$

$$\rho = \bar{\rho}(z) + \rho'(x, y, z, t)$$

$$\frac{\partial \bar{p}}{\partial z} = -\bar{\rho} g$$

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \Rightarrow \rho\frac{dw}{dt} = -\frac{\partial p}{\partial z} - \rho g$$

$$\rho\frac{dw}{dt} = -\frac{\partial \bar{p}}{\partial z} - \frac{\partial p'}{\partial z} - \bar{\rho} g - \rho' g$$

$$= -\frac{\partial p'}{\partial z} - \rho' g$$

中尺度和大尺度的动力学差别

$\frac{dw}{dt}$ 与 $-\frac{1}{\rho}\frac{\partial p}{\partial z}$ 尺度比较

$$\rho\frac{dw}{dt} = -\frac{\partial p'}{\partial z} - \rho' g \Rightarrow \frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p'}{\partial z} - \frac{\rho'}{\rho} g \quad \text{(E1.3)}$$

估计 $O(\frac{dw}{dt}) / O(-\frac{1}{\rho}\frac{\partial p\prime}{\partial z})$ 需要知道 $O(w)$ 和 $O(\frac{p\prime}{\rho})$

假定 $O(\frac{\partial w}{\partial z}) \sim O(\frac{\partial u}{\partial x})$

$$\frac{W}{H} \sim \frac{U}{L} \Rightarrow \frac{H}{W} \sim \frac{L}{U} \Rightarrow T_z \sim T_h \quad \text{(E1.4)}$$

垂直平流时间尺度（T_z）　水平平流时间尺度（T_h）

$$\Rightarrow W \sim \frac{UH}{L} \quad \text{(E1.5)} \qquad \Rightarrow O(\frac{dw}{dt}) \sim \frac{UH}{LT_z} \quad \text{(E1.6)}$$

中尺度和大尺度的动力学差别

$O(-\frac{1}{\rho}\frac{\partial p'}{\partial z}) \sim \frac{p'}{\rho H}$　　首先分析 $\frac{p'}{\rho}$

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv \approx -\frac{1}{\rho}\frac{\partial p}{\partial x} \approx -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} - \frac{1}{\rho}\frac{\partial p'}{\partial x} \approx -\frac{1}{\rho}\frac{\partial p'}{\partial x}$$

10^{-1}　　10^{-3}

$$O(\frac{du}{dt}) \sim O(-\frac{1}{\rho}\frac{\partial p'}{\partial x})$$

$$\frac{U}{T_h} \sim \frac{p'}{\rho L} \Rightarrow \frac{p'}{\rho} \sim \frac{UL}{T_h} \quad \text{(E1.8)}$$

$$O(-\frac{1}{\rho}\frac{\partial p'}{\partial z}) \sim \frac{UL}{HT_h} \quad \text{(E1.9)}$$

中尺度和大尺度的动力学差别

$\mathrm{T}_z\sim\mathrm{T}_h$

$$O\left(\frac{dw}{dt}\right)\Big/O\left(-\frac{1}{\rho}\frac{\partial p'}{\partial z}\right)\sim\frac{\mathrm{UH}}{\mathrm{LT}_z}\Big/\frac{\mathrm{UL}}{\mathrm{HT}_h}\sim\frac{\mathrm{UH}}{\mathrm{LT}_z}\cdot\frac{\mathrm{HT}_h}{\mathrm{UL}}\sim\left(\frac{\mathrm{H}}{\mathrm{L}}\right)^2$$

Aspect ratio

如果 $\frac{\mathrm{H}}{\mathrm{L}}\ll 1$，$\frac{dw}{dt}$ 可以忽略 $\Rightarrow$ 静力平衡

大尺度 $\frac{\mathrm{H}}{\mathrm{L}}\sim\frac{10\ \mathrm{km}}{1000\ \mathrm{km}}\sim 10^{-2}\ll 1$，满足静力平衡

中尺度 $\frac{\mathrm{H}}{\mathrm{L}}\sim\frac{10\ \mathrm{km}}{10\ \mathrm{km}}\sim 1$， 不满足静力平衡

并不是所有的中尺度系统都不满足静力平衡

比如：冷池 $\frac{\mathrm{H}}{\mathrm{L}}\sim\frac{2\ \mathrm{km}}{20\ \mathrm{km}}\sim 0.1$

主要内容

1.1 什么是中尺度天气系统

1.2 中尺度基本方程组

1.3 扰动气压

1.4 基本工具

Skew-T

Hodograph

Radar基础

简化原始方程

简单起见，不考虑摩擦和科氏力的垂直分量

$$
\begin{cases}
\dfrac{du}{dt} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x} + fv \\
\dfrac{dv}{dt} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial y} - fu \\
\dfrac{dw}{dt} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial z} + g \\
\dfrac{d\rho}{dt} + \rho\left(\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z}\right) = 0 \\
p = \rho RT \\
\dfrac{d\theta}{dt} = -\dfrac{\theta}{C_p T}\dfrac{dQ}{dt}
\end{cases}
$$

- 该方程不适合讨论中小尺度天气，因为方程中包含大、中、小各种尺度和声波，需要简化。
- 通过做合理假定，使方程中的某些项线性化，也可以使问题简化。

其他形式

$$\frac{dp}{dt} - C_s^2\frac{d\rho}{dt} = \frac{dQ}{dt}$$

$$C_s^2 = \gamma RT$$

$$C_v\frac{dT}{dt} + p\frac{d\alpha}{dt} = \frac{dQ}{dt}$$

$$\gamma = \frac{C_p}{C_v}$$

Boussinesq近似

$$\rho = \rho_0 + \delta\rho(x, y, z, t)$$

Summary of Boussinesq Equations

The simple Boussinesq equations are, for an inviscid fluid:

momentum equations: $$\frac{\mathrm{D}\boldsymbol{v}}{\mathrm{D}t} + \boldsymbol{f} \times \boldsymbol{v} = -\nabla\phi + b\mathbf{k}, \qquad \text{(B.1)}$$

mass conservation: $$\nabla \cdot \boldsymbol{v} = 0, \qquad \text{(B.2)}$$

buoyancy equation: $$\frac{\mathrm{D}b}{\mathrm{D}t} = \dot{b}. \qquad \text{(B.3)}$$

Anelastic近似

针对中尺度做简化处理:

（1）尺度分离 $f = \bar{f} + f'$

$\bar{f}$ 为大尺度参考量　　f' 为偏离 $\bar{f}$ 的中尺度扰动。

(2) 假定大尺度的时间变化远慢于中尺度扰动的时间变化。

定常

$$\left|\frac{\partial \bar{f}}{\partial t}\right| \ll \left|\frac{\partial f'}{\partial t}\right|$$

(3) 假定大尺度的水平梯度远小于中尺度扰动的水平梯度。

水平均匀

$$\left|\frac{\partial \bar{f}}{\partial x}\right| \ll \left|\frac{\partial f'}{\partial x}\right|, \quad \left|\frac{\partial \bar{f}}{\partial y}\right| \ll \left|\frac{\partial f'}{\partial y}\right|$$

(4) 假定天气尺度参考量 $\bar{f}$ 远大于中尺度扰动量 f'。 $\left|\frac{f'}{\bar{f}}\right| \ll 1$

(5) 大尺度背景满足静力平衡。 $$\frac{\partial \bar{p}}{\partial z} = -\bar{\rho} g$$

Anelastic近似

$$p = \bar{p}(z) + p'(x, y, z, t) \qquad \theta = \bar{\theta}(z) + \theta'(x, y, z, t)$$

$$\rho = \bar{\rho}(z) + \rho'(x, y, z, t) \qquad T = \bar{T}(z) + T'(x, y, z, t)$$

静态假定：$\bar{u} = 0, \bar{v} = 0, \bar{w} = 0$, 速度仅为扰动速度，简单起见，省略 ′， 记为$u, v, w$

运动方程

水平运动方程

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv$$

$$\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu$$

$$\frac{du}{dt} = -\frac{1}{\bar{\rho} + \rho'}\frac{\partial(\bar{p} + p')}{\partial x} + fv$$

$$= -\frac{1}{\bar{\rho} + \rho'}\frac{\partial p'}{\partial x} + fv$$

$$\frac{du}{dt} \approx -\frac{1}{\bar{\rho}}\frac{\partial p'}{\partial x} + fv$$

$$\frac{dv}{dt} \approx -\frac{1}{\bar{\rho}}\frac{\partial p'}{\partial y} - fu$$

运动方程

垂直运动方程

$$\frac{dw}{dt} = -\frac{1}{\bar{\rho} + \rho'} \left(\frac{\partial \bar{p}}{\partial z} + \frac{\partial p'}{\partial z}\right) - g$$

$$= -\frac{1}{\bar{\rho} + \rho'} \left(-\bar{\rho} g + \frac{\partial p'}{\partial z}\right) - g$$

$$= -\frac{1}{\bar{\rho} + \rho'} \frac{\partial p'}{\partial z} + \left(\frac{\bar{\rho}}{\bar{\rho} + \rho'} - 1\right) g$$

$$= -\frac{1}{\bar{\rho} + \rho'} \frac{\partial p'}{\partial z} - \frac{\rho'}{\bar{\rho} + \rho'} g$$

$$\approx -\frac{1}{\bar{\rho}} \frac{\partial p'}{\partial z} - \frac{\rho'}{\bar{\rho}} g$$

由密度扰动引起的浮力

Class Break

连续方程

$$\frac{\partial\rho}{\partial t}=-(u\frac{\partial\rho}{\partial x}+v\frac{\partial\rho}{\partial y}+w\frac{\partial\rho}{\partial z})\quad-\rho\,(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z})$$

比容 $\frac{\partial\alpha}{\partial t}=-\left(u\frac{\partial\alpha}{\partial x}+v\frac{\partial\alpha}{\partial y}+w\frac{\partial\alpha}{\partial z}\right)+\alpha(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z})$

令 $\alpha=\bar{\alpha}(z)+\alpha'(x,y,z,t)$

$$\frac{\partial\bar{\alpha}}{\partial t}+\frac{\partial\alpha'}{\partial t}=-\left(u\frac{\partial\alpha'}{\partial x}+v\frac{\partial\alpha'}{\partial y}+w\frac{\partial\alpha'}{\partial z}+u\frac{\partial\bar{\alpha}}{\partial x}+v\frac{\partial\bar{\alpha}}{\partial y}+w\frac{\partial\bar{\alpha}}{\partial z}\right)$$

$$+\bar{\alpha}(1+\frac{\alpha'}{\bar{\alpha}})\;(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z})$$

根据（2）-（4）的假定

连续方程

$$\frac{\partial \alpha'}{\partial t} \approx -\left[\left(u\frac{\partial \alpha'}{\partial x} + v\frac{\partial \alpha'}{\partial y} + w\frac{\partial \alpha'}{\partial z}\right) + w\frac{\partial \bar{\alpha}}{\partial z}\right] + \bar{\alpha}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

特征尺度：x，y~L, z~H, u, v~U, $t\sim\frac{\text{L}}{\text{U}}$, $w\sim\frac{\text{UH}}{\text{L}}$

假定：$\frac{\partial w}{\partial z} \sim \frac{\partial u}{\partial x} \sim \frac{\text{U}}{\text{L}}$

下面把每一项与$\bar{\alpha}\frac{\partial w}{\partial z}$相比

$$\left|\frac{\partial \alpha'}{\partial t}\right| / \left|\bar{\alpha}\frac{\partial w}{\partial z}\right| \sim \frac{\alpha'}{\bar{\alpha}} \cdot \frac{\text{U}}{\text{L}} \cdot \frac{\text{L}}{\text{U}} = \frac{\alpha'}{\bar{\alpha}} \ll 1$$

$$\left|u\frac{\partial \alpha'}{\partial x}\right| / \left|\bar{\alpha}\frac{\partial w}{\partial z}\right| \sim \frac{\alpha'}{\bar{\alpha}} \cdot \frac{\text{U}}{\text{L}} \cdot \frac{\text{L}}{\text{U}} = \frac{\alpha'}{\bar{\alpha}} \ll 1$$

连续方程

$$\frac{\partial \alpha'}{\partial t} \approx -\left[\left(u\frac{\partial \alpha'}{\partial x} + v\frac{\partial \alpha'}{\partial y} + w\frac{\partial \alpha'}{\partial z}\right) + w\frac{\partial \overline{\alpha}}{\partial z}\right] + \overline{\alpha}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

$$\left|v\frac{\partial \alpha'}{\partial y}\right| / \left|\overline{\alpha}\frac{\partial w}{\partial z}\right| \sim \frac{\alpha'}{\overline{\alpha}} \cdot \frac{\mathrm{U}}{\mathrm{L}} \cdot \frac{\mathrm{L}}{\mathrm{U}} = \frac{\alpha'}{\overline{\alpha}} \ll 1$$

$$\left|w\frac{\partial \alpha'}{\partial z}\right| / \left|\overline{\alpha}\frac{\partial w}{\partial z}\right| \sim \frac{\mathrm{UH}}{\mathrm{L}} \cdot \frac{\alpha'}{\mathrm{H}} \cdot \frac{1}{\overline{\alpha}} \cdot \frac{\mathrm{L}}{\mathrm{U}} = \frac{\alpha'}{\overline{\alpha}} \ll 1$$

对于α'，$\alpha' \approx \partial\alpha'$

$$\left|w\frac{\partial \overline{\alpha}}{\partial z}\right| / \left|\overline{\alpha}\frac{\partial w}{\partial z}\right| \sim \frac{\mathrm{UH}}{\mathrm{L}} \cdot \frac{\mathrm{L}}{\mathrm{U}} / \left|\frac{1}{\overline{\alpha}}\frac{\partial \overline{\alpha}}{\partial z}\right|^{-1} = \mathrm{H} / \left|\frac{1}{\overline{\alpha}}\frac{\partial \overline{\alpha}}{\partial z}\right|^{-1} = \frac{\mathrm{H}}{\mathrm{H}_\alpha}$$

对于$\overline{\alpha}$，$\overline{\alpha} \neq \partial\overline{\alpha}$, 对于不同的H, $\partial\overline{\alpha}$不一样

标高: 大气密度减小到起始密度的$1/e$时的高度增量 $\mathrm{H}_\alpha \sim \left|\frac{1}{\overline{\alpha}}\frac{\partial \overline{\alpha}}{\partial z}\right|^{-1}$

连续方程

$$\frac{\partial \alpha'}{\partial t} \approx -\left[\left(u\frac{\partial \alpha'}{\partial x} + v\frac{\partial \alpha'}{\partial y} + w\frac{\partial \alpha'}{\partial z}\right) + w\frac{\partial \overline{\alpha}}{\partial z}\right] + \overline{\alpha}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \boxed{\frac{\partial w}{\partial z}}\right)$$

$$\left|\overline{\alpha}\frac{\partial u}{\partial x}\right| / \left|\overline{\alpha}\frac{\partial w}{\partial z}\right| \sim \frac{\mathrm{U}}{\mathrm{L}}\cdot\frac{\mathrm{L}}{\mathrm{U}} = 1 \qquad \left|\overline{\alpha}\frac{\partial v}{\partial y}\right| / \left|\overline{\alpha}\frac{\partial w}{\partial z}\right| \sim \frac{\mathrm{U}}{\mathrm{L}}\cdot\frac{\mathrm{L}}{\mathrm{U}} = 1$$

如果 $\frac{\mathrm{H}}{\mathrm{H}_\alpha} \ll 1$, $\overline{\alpha}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$ 或 $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

$$\nabla \cdot \vec{\boldsymbol{v}} = 0$$

运动的垂直尺度远小于大气密度标高的情况下的不可压缩近似，或Boussinesq近似，为浅对流连续方程，滤掉了声波（$\frac{\partial \rho'}{\partial t} = 0$）

连续方程

$$\frac{\partial \alpha'}{\partial t} \approx -\left[\left(u\frac{\partial \alpha'}{\partial x} + v\frac{\partial \alpha'}{\partial y} + w\frac{\partial \alpha'}{\partial z}\right) + w\frac{\partial \overline{\alpha}}{\partial z}\right] + \overline{\alpha}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

如果 $\frac{H}{H_\alpha}$ ≪1不成立, 则有: $-w\frac{\partial \overline{\alpha}}{\partial z} + \overline{\alpha}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$

$$w\frac{\partial \overline{\rho}}{\partial z} + \overline{\rho}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$$

$$u\frac{\partial \overline{\rho}}{\partial x} + v\frac{\partial \overline{\rho}}{\partial y} + w\frac{\partial \overline{\rho}}{\partial z} + \overline{\rho}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$$

$$\frac{\partial u\overline{\rho}}{\partial x} + \frac{\partial v\overline{\rho}}{\partial y} + \frac{\partial w\overline{\rho}}{\partial z} = 0$$

$\nabla \cdot \overline{\rho}\vec{\boldsymbol{v}} = 0$ 滞弹性近似 (Anelastic Approximation)

状态方程

p=ρRT $\Rightarrow$ $\bar{p}+p'=(\bar{\rho}+\rho')R(\bar{T}+T')$

$\bar{p}(1+\frac{p'}{\bar{p}})=\bar{\rho}(1+\frac{\rho'}{\bar{\rho}})R\bar{T}(1+\frac{T'}{\bar{T}})$

两边取对数

$$\ln\bar{p}+\ln(1+\frac{p'}{\bar{p}})=\ln\bar{\rho}+\ln\left(1+\frac{\rho'}{\bar{\rho}}\right)+\ln R\bar{T}+\ln\left(1+\frac{T'}{\bar{T}}\right)$$

由于 $\ln\bar{p}=\ln\bar{\rho}+\ln R\bar{T}$

$$\Rightarrow\ \ln(1+\frac{p'}{\bar{p}})=\ln\left(1+\frac{\rho'}{\bar{\rho}}\right)+\ln\left(1+\frac{T'}{\bar{T}}\right)$$

状态方程

由于
$$\begin{cases} \ln(1+\frac{p'}{\bar{p}}) \approx \frac{p'}{\bar{p}} \\ \ln(1+\frac{\rho'}{\bar{\rho}}) \approx \frac{\rho'}{\bar{\rho}} \\ \ln(1+\frac{T'}{\bar{T}}) \approx \frac{T'}{\bar{T}} \end{cases} \Rightarrow \quad \frac{p'}{\bar{p}} \approx \frac{\rho'}{\bar{\rho}} + \frac{T'}{\bar{T}}$$

观测证明，大多数中小尺度系统均满足 $\frac{p'}{\bar{p}} \ll \frac{T'}{\bar{T}}$

$$\Rightarrow \quad -\frac{\rho'}{\bar{\rho}} \approx \frac{T'}{\bar{T}}$$

表示浮力项主要由温度扰动引起，浮力可以通过温度差计算。

热力学方程

$$\frac{1}{T}\frac{dQ}{dt} = c_p \frac{d\ln\theta}{dt}$$

绝热条件下 $\frac{d\ln\theta}{dt} = 0,$ 令 $\theta = \bar{\theta} + \theta'$

$$\Rightarrow \quad \frac{d\ln(\bar{\theta} + \theta')}{dt} = 0$$

$$\frac{1}{\bar{\theta} + \theta'}\frac{d(\bar{\theta} + \theta')}{dt} = 0$$

$$\frac{1}{\bar{\theta}}\left(\frac{d\bar{\theta}}{dt} + \frac{d\theta'}{dt}\right) = 0$$

热力学方程

$$\frac{1}{\bar{\theta}}\frac{d\theta'}{dt}+\frac{1}{\bar{\theta}}\left(\frac{\partial\bar{\theta}}{\partial t}+u\frac{\partial\bar{\theta}}{\partial x}+v\frac{\partial\bar{\theta}}{\partial y}+w\frac{\partial\bar{\theta}}{\partial z}\right)=0$$

$$\frac{1}{\bar{\theta}}\frac{d\theta'}{dt}+\frac{w}{\bar{\theta}}\frac{\partial\bar{\theta}}{\partial z}=0 \quad\Rightarrow\quad \frac{d\theta'}{dt}=-w\frac{\partial\bar{\theta}}{\partial z}$$

可用于稳定度分析

令 $s=\frac{1}{\bar{\theta}}\frac{\partial\bar{\theta}}{\partial z}$，可得

$$\frac{1}{\bar{\theta}}\frac{d\theta'}{dt}+sw=0$$

中小尺度天气方程组

$$
\begin{cases}
\dfrac{du}{dt} = -\dfrac{1}{\bar{\rho}}\dfrac{\partial p'}{\partial x} + fv \\
\dfrac{dv}{dt} = -\dfrac{1}{\bar{\rho}}\dfrac{\partial p'}{\partial y} - fu \\
\dfrac{dw}{dt} = -\dfrac{1}{\bar{\rho}}\dfrac{\partial p'}{\partial z} - \dfrac{\rho'}{\bar{\rho}}g \\
\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z} = 0 \\
\dfrac{p'}{\bar{p}} = \dfrac{\rho'}{\bar{\rho}} + \dfrac{T'}{\bar{T}} \quad \text{或} \quad -\dfrac{\rho'}{\bar{\rho}} \approx \dfrac{T'}{\bar{T}} \\
\dfrac{d\theta'}{dt} = -w\dfrac{\partial \bar{\theta}}{\partial z}
\end{cases}
$$

适用于发生在浅层内的中尺度运动：积云对流、海陆风、边界层急流中的重力波等。

Anelastic近似方程组

$$
\boxed{
\begin{aligned}
\frac{\mathrm{D}\boldsymbol{v}}{\mathrm{D}t} + 2\boldsymbol{\Omega} \times \boldsymbol{v} &= \mathbf{k}b_a - \nabla\phi \\
\frac{\mathrm{D}b_a}{\mathrm{D}t} &= 0 \\
\nabla \cdot (\tilde{\rho}\boldsymbol{v}) &= 0
\end{aligned}
}\,,
$$