中尺度的特殊性

a. 基本方程组最接近于原始方程

大尺度: $\frac{dw}{dt}$, $\vec{v_a}\cdot\nabla\alpha$ 可以忽略

小尺度: 科氏力，甚至水平气压梯度力可以忽略

$$\begin{cases} \frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv \\ \frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu \\ \frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + g \\ \frac{d\rho}{dt} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0 \\ p = \rho RT \\ \frac{d\theta}{dt} = -\frac{\theta}{C_pT}\frac{dQ}{dt} \end{cases}$$

b. 运动受多种不稳定性控制

大尺度: 斜压不稳定

中尺度: 静力不稳定，对称不稳定，正压不稳定（Kelvin-Helmholtz不稳定）

小尺度: 静力不稳定

c. 时间尺度

大尺度: 惯性震荡 ($\frac{2\pi}{f}$ ~17 h)

小尺度: 纯浮力震荡 ($\frac{2\pi}{N}$ ~ 10 min)

简化原始方程

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv$$

$$\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu$$

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + g$$

$$\frac{d\rho}{dt} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$$

$$p = \rho RT$$

$$\frac{d\theta}{dt} = -\frac{\theta}{C_p T}\frac{dQ}{dt}$$

Anelastic近似, 滤去声波

$$\frac{D\boldsymbol{v}}{Dt} + 2\boldsymbol{\Omega} \times \boldsymbol{v} = \mathbf{k} b_a - \nabla\phi$$

$$\frac{Db_a}{Dt} = 0$$

$$\nabla \cdot (\tilde{\rho}\boldsymbol{v}) = 0 ,$$

简单起见，不考虑摩擦和科氏力的垂直分量

中尺度和大尺度的动力学差别

T_z~T_h

$$O\left(\frac{dw}{dt}\right)\Big/O\left(-\frac{1}{\rho}\frac{\partial p'}{\partial z}\right)\sim\frac{UH}{LT_z}\Big/\frac{UL}{HT_h}\sim\frac{UH}{LT_z}\cdot\frac{HT_h}{UL}\sim\left(\frac{H}{L}\right)^2$$

Aspect ratio

如果 $\frac{H}{L}\ll 1$，$\frac{dw}{dt}$可以忽略 $\Rightarrow$ 静力平衡

大尺度 $\frac{H}{L}\sim\frac{10\text{ km}}{1000\text{ km}}\sim 10^{-2}\ll 1$， 满足静力平衡

中尺度 $\frac{H}{L}\sim\frac{10\text{ km}}{10\text{ km}}\sim 1$， 不满足静力平衡

并不是所有的中尺度系统都不满足静力平衡

比如：冷池 $\frac{H}{L}\sim\frac{2\text{ km}}{20\text{ km}}\sim 0.1$

尺度分析

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv$$

天气尺度:

见板书

中尺度:

主要内容

水平尺度小于Rossby变形半径时，气压场向风场适应，可**通过风和温度来估计气压**，并用于理解影响中尺度现象的结构和演变的强迫因子。

(0) 静力平衡大气中的气压变化: Z坐标

a. 高度坐标系 $$\frac{\partial p}{\partial z} = -\rho g$$

$$p(z) = g\int_z^{\infty} \rho dz$$

某一高度上气压为该高度以上单位面积内大气的重量

$$\frac{\partial p}{\partial t} = g\int_z^{\infty} \frac{\partial \rho}{\partial t} dz$$

把连续方程代入，可得

$$\frac{d\rho}{dt} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$$

$$= g\int_z^{\infty} \left(-\frac{\partial \rho u}{\partial x} - \frac{\partial \rho v}{\partial y} - \frac{\partial \rho w}{\partial z}\right) dz$$

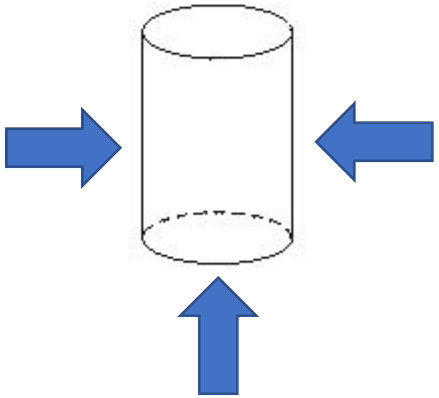
$$= g\int_z^{\infty} \left(-\frac{\partial \rho u}{\partial x} - \frac{\partial \rho v}{\partial y}\right) dz - g\int_z^{\infty} d(\rho w)$$

(0) 静力平衡大气中的气压变化: Z坐标

在 $z=\infty$ 时，$\rho=0 \Rightarrow \rho w=0$，于是

$$\frac{\partial p}{\partial t}=g\int_{z}^{\infty}\left(-\frac{\partial \rho u}{\partial x}-\frac{\partial \rho v}{\partial y}\right)dz+g(\rho w)|_{z}$$

净质量辐合　　z处的垂直平流



（1）静力平衡大气中的气压变化: p坐标

a. 气压坐标系 $\frac{\partial p}{\partial z} = -\rho g$

$$\Rightarrow \quad \frac{\partial z}{\partial p} = -\frac{RT}{gp} \qquad \frac{\partial \phi}{\partial p} = -\frac{RT}{p}$$

$$\partial z = -\frac{RT}{gp}\partial p = -\frac{RT}{g}\partial \ln p$$

$$z(p_b) - z(p_t) = -\int_{p_t}^{p_b} \frac{RT}{g} d \ln p$$

两边对时间求偏导数

(1) 静力平衡大气中的气压变化: p坐标

$$\frac{\partial z(p_b)}{\partial t} - \frac{\partial z(p_t)}{\partial t} = -\frac{1}{g}\int_{p_t}^{p_b} \frac{\partial RT}{\partial t}\, d\ln p$$

$$= -\frac{1}{g}\int_{p_t}^{p_b} \frac{R\partial T}{\partial t}\, d\ln p \qquad \text{(E3.1)}$$

温度随时间的变化是由什么过程决定?

由热力学方程 $c_v \frac{dT}{dt} + p\frac{d\alpha}{dt} = \frac{dQ}{dt} = q$

$$c_v \frac{dT}{dt} + p\frac{d\frac{1}{\rho}}{dt} = q \Rightarrow c_v \frac{dT}{dt} + p\frac{d\frac{RT}{p}}{dt} = q$$

(1) 静力平衡大气中的气压变化: p坐标

$$c_v \frac{dT}{dt} + p\left(-\frac{RT}{p^2}\frac{dp}{dt} + \frac{R}{p}\frac{dT}{dt}\right) = q$$ 括号展开

$$c_v \frac{dT}{dt} + R\frac{dT}{dt} - \frac{RT}{p}\frac{dp}{dt} = q$$

由于 $c_v + R = c_p$,气压坐标系下 $\frac{dp}{dt} = \omega$

$$\Rightarrow \quad c_p \frac{dT}{dt} - \frac{RT}{p}\omega = q \quad \Rightarrow \quad \frac{dT}{dt} - \frac{RT}{c_p p}\omega = \frac{q}{c_p}$$

$$\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla_p T + \omega\left(\frac{\partial T}{\partial p} - \frac{RT}{c_p p}\right) = \frac{q}{c_p}$$

(1) 静力平衡大气中的气压变化: p坐标

$$\frac{\partial T}{\partial t}+\vec{\mathrm{v}}\cdot\nabla_p T+\omega\left(\frac{\partial T}{\partial p}-\frac{\mathrm{R}T}{\mathrm{c_p}p}\right)=\frac{q}{\mathrm{c_p}}$$

括号里的项如何更简洁地表示为物理过程?

(E3.2)

$$\theta=T\left(\frac{1000}{p}\right)^{\mathrm{R}/\mathrm{c_p}}\Rightarrow\ln\theta=\ln T+\frac{\mathrm{R}}{\mathrm{c_p}}(\ln 1000-\ln p)$$

$$\frac{\partial\ln\theta}{\partial p}=\frac{\partial\ln T}{\partial p}-\frac{\mathrm{R}}{\mathrm{c_p}p}=\frac{1}{T}\frac{\partial T}{\partial p}-\frac{\mathrm{R}}{\mathrm{c_p}p}$$

$$T\frac{\partial\ln\theta}{\partial p}=\frac{\partial T}{\partial p}-\frac{\mathrm{R}T}{\mathrm{c_p}p}=\frac{T}{\theta}\frac{\partial\theta}{\partial p}=-\sigma$$

引入稳定度参数 $\sigma=-\dfrac{T}{\theta}\dfrac{\partial\theta}{\partial p}$

$\sigma>0$, 稳定

$\sigma<0$, 不稳定

（1）静力平衡大气中的气压变化: p坐标

把 $\frac{\partial T}{\partial p}-\frac{RT}{c_p p}=-\sigma$ 代入 E3.2 $\frac{\partial T}{\partial t}+\vec{v}\cdot\nabla_p T+\omega\left(\frac{\partial T}{\partial p}-\frac{RT}{c_p p}\right)=\frac{q}{c_p}$

$$\Rightarrow \frac{\partial T}{\partial t}=-\vec{v}\cdot\nabla_p T+\omega\sigma+\frac{q}{c_p}$$

代入 E3.1

（E3.1） $$\frac{\partial z(p_b)}{\partial t}-\frac{\partial z(p_t)}{\partial t}=-\frac{1}{g}\int_{p_t}^{p_b}\frac{R\partial T}{\partial t}\,d\ln p$$

$$\frac{\partial z(p_b)}{\partial t}-\frac{\partial z(p_t)}{\partial t}=-\frac{1}{g}\int_{p_t}^{p_b}R\left(-\vec{v}\cdot\nabla_p T+\omega\sigma+\frac{q}{c_p}\right)d\ln p$$

$$\frac{\partial z(p_b)}{\partial t}=-\frac{1}{g}\int_{p_t}^{p_b}R\left(-\vec{v}\cdot\nabla_p T+\omega\sigma+\frac{q}{c_p}\right)d\ln p+\frac{\partial z(p_t)}{\partial t}$$

(1) 静力平衡大气中的气压变化: p坐标

$$\frac{\partial z(p_b)}{\partial t} = -\frac{1}{\mathrm{g}}\int_{p_t}^{p_b} \mathrm{R}\left(-\vec{\mathrm{v}}\cdot\nabla_p T + \omega\sigma + \frac{q}{\mathrm{c_p}}\right) d\ln p + \frac{\partial z(p_t)}{\partial t}$$

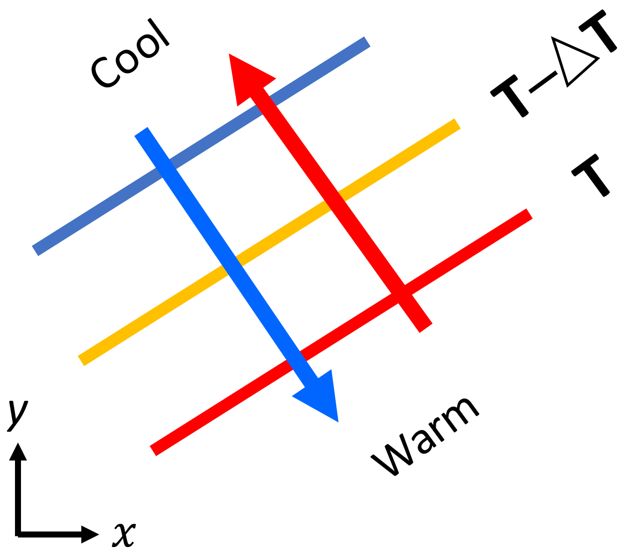
温度平流　　绝热过程　　非绝热过程

p_t很小时，约为0

地面

< 0，减低　　暖平流

> 0，增压　　冷平流



(1) 静力平衡大气中的气压变化: p坐标

$$\frac{\partial z(p_b)}{\partial t} = -\frac{1}{\mathrm{g}}\int_{p_t}^{p_b} \mathrm{R}\left(-\vec{\mathrm{v}}\cdot\nabla_p T + \omega\sigma + \frac{q}{\mathrm{c_p}}\right) d\ln p + \frac{\partial z(p_t)}{\partial t}$$

温度平流　　绝热过程　　非绝热过程

p_t很小时，约为0

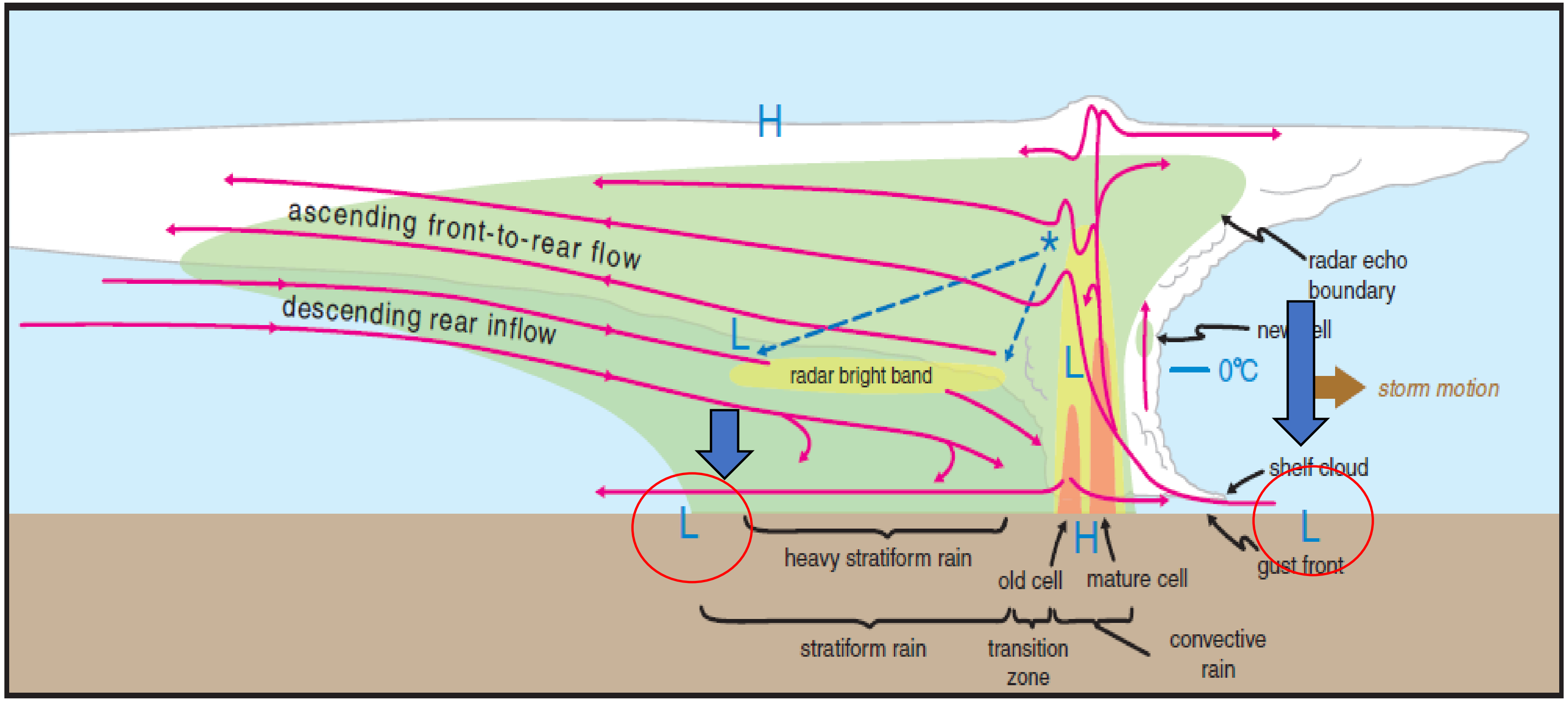
对于稳定情形 $\sigma > 0$

地面

< 0，减低　　暖平流　　$\omega > 0$ 绝热下沉

> 0，增压　　冷平流　　$\omega < 0$ 绝热上升

（1）静力平衡大气中的气压变化: p坐标



(1) 静力平衡大气中的气压变化: p坐标

$$\frac{\partial z(p_b)}{\partial t} = -\frac{1}{\mathrm{g}}\int_{p_t}^{p_b} \mathrm{R}\left(-\vec{\mathrm{v}}\cdot\nabla_p T + \omega\sigma + \frac{q}{\mathrm{c_p}}\right) d\ln p + \frac{\partial z(p_t)}{\partial t}$$

温度平流　　绝热过程　　非绝热过程

p_t很小时，约为0

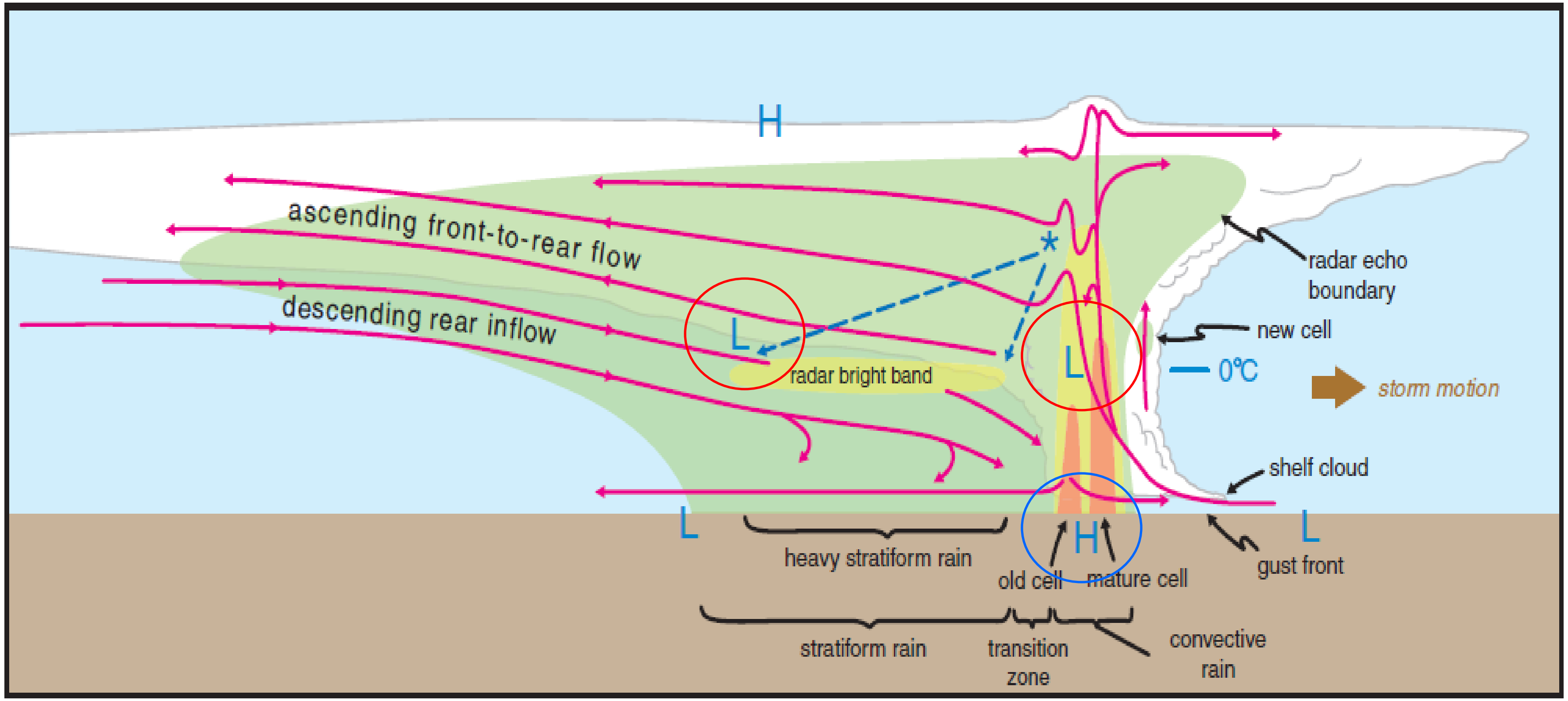
对于稳定情形 $\sigma > 0$

地面

< 0，减低　　暖平流　　$\omega > 0$ 绝热下沉　　非绝热加热　　辐射加热 凝结

> 0，增压　　冷平流　　$\omega < 0$ 绝热上升　　非绝热冷却　　辐射冷却 蒸发

（1）静力平衡大气中的气压变化: p坐标



(2) 静力和非静力扰动气压

$$p = \bar{p}(z) + p' \qquad \frac{\partial \bar{p}}{\partial z} = -\bar{\rho} g$$

$$\frac{dw}{dt} = -\frac{1}{\bar{\rho}}\frac{\partial p'}{\partial z} - \frac{\rho'}{\bar{\rho}} g$$

静力条件下，垂直加速度为0. 扰动气压来源于扰动密度

$$\frac{\partial p'_{\mathrm{h}}}{\partial z} = -\rho' g$$

(2) 静力和非静力扰动气压

非静力条件下，气压的变化无法准确地由积分静力方程得到

$$p' = p'_{\mathrm{h}} + p'_{\mathrm{nh}}$$

$$\frac{dw}{dt} = -\frac{1}{\bar{\rho}}\frac{\partial p'}{\partial z} - \frac{\rho'}{\bar{\rho}}g = -\frac{1}{\bar{\rho}}\frac{\partial p'_{\mathrm{h}}}{\partial z} - \frac{1}{\bar{\rho}}\frac{\partial p'_{\mathrm{nh}}}{\partial z} - \frac{\rho'}{\bar{\rho}}g$$

$$\Rightarrow \frac{dw}{dt} = -\frac{1}{\bar{\rho}}\frac{\partial p'_{\mathrm{nh}}}{\partial z}$$

非静力扰动气压产生垂直加速度

(3) 动力和浮力扰动气压

a. 扰动气压诊断方程 （简单起见，使用Boussinesq近似）

$$\frac{\partial \vec{\boldsymbol{v}}}{\partial t}+\vec{\boldsymbol{v}}\cdot\nabla\vec{\boldsymbol{v}}=-\alpha_0\nabla\boldsymbol{p}'+B\vec{\mathrm{k}}-f\vec{\mathrm{k}}\times\vec{\boldsymbol{v}}$$

$$B=-\frac{\rho'}{\rho_0}g$$

$$\alpha_0\equiv\frac{1}{\rho_0}$$

$$\begin{cases}\dfrac{\partial u}{\partial t}+u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}+w\dfrac{\partial u}{\partial z}=-\alpha_0\dfrac{\partial p'}{\partial x}+fv & ①\\[2ex] \dfrac{\partial v}{\partial t}+u\dfrac{\partial v}{\partial x}+v\dfrac{\partial v}{\partial y}+w\dfrac{\partial v}{\partial z}=-\alpha_0\dfrac{\partial p'}{\partial y}-fu & ②\\[2ex] \dfrac{\partial w}{\partial t}+u\dfrac{\partial w}{\partial x}+v\dfrac{\partial w}{\partial y}+w\dfrac{\partial w}{\partial z}=-\alpha_0\dfrac{\partial p'}{\partial z}+B & ③\end{cases}$$

我们要得到扰动气压诊断方程,要尽量去掉其他项。

(3) 动力和浮力扰动气压

考虑Boussinesq近似: $\nabla \cdot \vec{\boldsymbol{v}} = 0$ $\quad \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\alpha_0\frac{\partial p'}{\partial x} + fv \quad ①$$

$$\frac{\partial ①}{\partial x}: \quad \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right) + \left(\frac{\partial u}{\partial x}\right)^2 + u\frac{\partial^2 u}{\partial x^2} + \frac{\partial v}{\partial x}\frac{\partial u}{\partial y} + v\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial w}{\partial x}\frac{\partial u}{\partial z} + w\frac{\partial^2 u}{\partial z \partial x} = -\alpha_0\frac{\partial^2 p'}{\partial x^2} + f\frac{\partial v}{\partial x}$$

(3) 动力和浮力扰动气压

考虑Boussinesq近似： $\nabla \cdot \vec{\boldsymbol{v}} = 0 \quad \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\alpha_0\frac{\partial p'}{\partial y} - fu \quad ②$$

$$\frac{\partial ②}{\partial y}: \quad \frac{\partial}{\partial t}\left(\frac{\partial v}{\partial y}\right) + \frac{\partial u}{\partial y}\frac{\partial v}{\partial x} + u\frac{\partial^2 v}{\partial x \partial y} + \left(\frac{\partial v}{\partial y}\right)^2 + v\frac{\partial^2 v}{\partial y^2} + \frac{\partial w}{\partial y}\frac{\partial v}{\partial z} + w\frac{\partial^2 v}{\partial z \partial y} = -\alpha_0\frac{\partial^2 p'}{\partial y^2} - f\frac{\partial u}{\partial y} - \beta u$$

$$\beta = \frac{\partial f}{\partial y}$$

(3) 动力和浮力扰动气压

考虑Boussinesq近似： $\nabla\cdot\vec{\boldsymbol{v}}=0$ $\quad \frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\alpha_0\frac{\partial p'}{\partial z}+B \quad ③$$

$\frac{\partial ③}{\partial z}$:

$$\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right)+\frac{\partial u}{\partial z}\frac{\partial w}{\partial x}+u\frac{\partial^2 w}{\partial x\partial z}+\frac{\partial v}{\partial z}\frac{\partial w}{\partial y}+v\frac{\partial^2 w}{\partial y\partial z}+\left(\frac{\partial w}{\partial z}\right)^2+w\frac{\partial^2 w}{\partial z^2}=-\alpha_0\frac{\partial^2 p'}{\partial z^2}+\frac{\partial B}{\partial z}$$

$\frac{\partial ①}{\partial x}+\frac{\partial ②}{\partial y}+\frac{\partial ③}{\partial z}$， 利用 $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$ 假定f为常数

(3) 动力和浮力扰动气压

$$\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right)+\left(\frac{\partial u}{\partial x}\right)^2+u\frac{\partial^2 u}{\partial x^2}+\frac{\partial v}{\partial x}\frac{\partial u}{\partial y}+v\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial w}{\partial x}\frac{\partial u}{\partial z}+w\frac{\partial^2 u}{\partial z\partial x}=-\alpha_0\frac{\partial^2 p'}{\partial x^2}+f\frac{\partial v}{\partial x}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial y}\right)+\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}+u\frac{\partial^2 v}{\partial x\partial y}+\left(\frac{\partial v}{\partial y}\right)^2+v\frac{\partial^2 v}{\partial y^2}+\frac{\partial w}{\partial y}\frac{\partial v}{\partial z}+w\frac{\partial^2 v}{\partial z\partial y}=-\alpha_0\frac{\partial^2 p'}{\partial y^2}-f\frac{\partial u}{\partial y}-\beta u$$

$$\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right)+\frac{\partial u}{\partial z}\frac{\partial w}{\partial x}+u\frac{\partial^2 w}{\partial x\partial z}+\frac{\partial v}{\partial z}\frac{\partial w}{\partial y}+v\frac{\partial^2 w}{\partial y\partial z}+\left(\frac{\partial w}{\partial z}\right)^2+w\frac{\partial^2 w}{\partial z^2}=-\alpha_0\frac{\partial^2 p'}{\partial z^2}+\frac{\partial B}{\partial z}$$

（3）动力和浮力扰动气压

$$+\left(\frac{\partial u}{\partial x}\right)^2 \qquad + \frac{\partial v}{\partial x}\frac{\partial u}{\partial y}$$

$$+ \frac{\partial w}{\partial x}\frac{\partial u}{\partial z} \qquad = -\alpha_0 \frac{\partial^2 p'}{\partial x^2} + f\frac{\partial v}{\partial x}$$

$$+ \frac{\partial u}{\partial y}\frac{\partial v}{\partial x} \qquad +\left(\frac{\partial v}{\partial y}\right)^2$$

$$+ \frac{\partial w}{\partial y}\frac{\partial v}{\partial z} \qquad = -\alpha_0 \frac{\partial^2 p'}{\partial y^2} - f\frac{\partial u}{\partial y}$$

$$+ \frac{\partial u}{\partial z}\frac{\partial w}{\partial x} \qquad + \frac{\partial v}{\partial z}\frac{\partial w}{\partial y}$$

$$+\left(\frac{\partial w}{\partial z}\right)^2 \qquad = -\alpha_0 \frac{\partial^2 p'}{\partial z^2} + \frac{\partial B}{\partial z}$$

（3）动力和浮力扰动气压

$$\alpha_0\left(\frac{\partial^2 p'}{\partial x^2}+\frac{\partial^2 p'}{\partial y^2}+\frac{\partial^2 p'}{\partial z^2}\right)=-\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right]$$

$$-2\left[\frac{\partial v}{\partial x}\frac{\partial u}{\partial y}+\frac{\partial w}{\partial x}\frac{\partial u}{\partial z}+\frac{\partial w}{\partial y}\frac{\partial v}{\partial z}\right]+\frac{\partial B}{\partial z}+\mathrm{f}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$$

$$\nabla^2 p'=\frac{\partial^2 p'}{\partial x^2}+\frac{\partial^2 p'}{\partial y^2}+\frac{\partial^2 p'}{\partial z^2}$$

$$\zeta=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$$

$$\alpha_0\,\nabla^2 p'=-\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right]-2\left[\frac{\partial v}{\partial x}\frac{\partial u}{\partial y}+\frac{\partial w}{\partial x}\frac{\partial u}{\partial z}+\frac{\partial w}{\partial y}\frac{\partial v}{\partial z}\right]+\frac{\partial B}{\partial z}+\mathrm{f}\zeta$$

$$\nabla^2 p'=-\rho_0\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right]-2\rho_0\left[\frac{\partial v}{\partial x}\frac{\partial u}{\partial y}+\frac{\partial w}{\partial x}\frac{\partial u}{\partial z}+\frac{\partial w}{\partial y}\frac{\partial v}{\partial z}\right]+\rho_0\frac{\partial B}{\partial z}+\mathrm{f}\rho_0\zeta$$

(3) 动力和浮力扰动气压

$$\nabla^2 p' = -\rho_0\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] - 2\rho_0\left[\frac{\partial v}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial u}{\partial z} + \frac{\partial w}{\partial y}\frac{\partial v}{\partial z}\right] + \rho_0\frac{\partial B}{\partial z} + \mathrm{f}\rho_0\zeta$$

矢量形式 $\nabla^2 p' = -\rho_0 \nabla\cdot(\vec{\boldsymbol{v}}\cdot\nabla\vec{\boldsymbol{v}}) + \rho_0\frac{\partial B}{\partial z} - \rho_0 \mathrm{f}\, \nabla\cdot(\vec{\mathrm{k}}\times\vec{\boldsymbol{v}})$

$\nabla^2 p_{\mathrm{D}}'$	$\nabla^2 p_{\mathrm{B}}'$	$\nabla^2 p_{\mathrm{G}}'$
动力	浮力	地转

几点说明:

（1）适用于anelastic或Boussinesq近似;

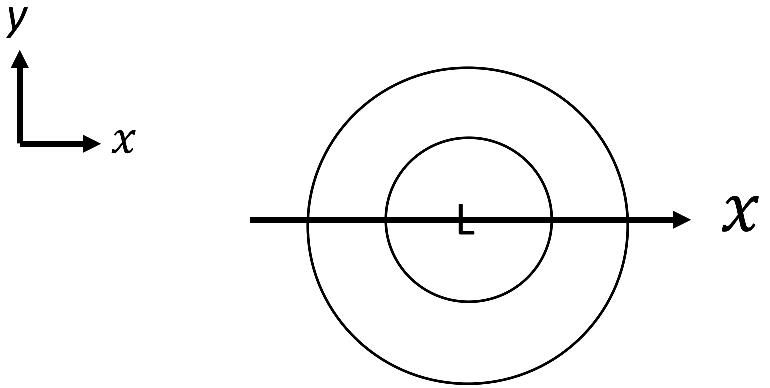
（2）可以基于风场和浮力场计算气压;

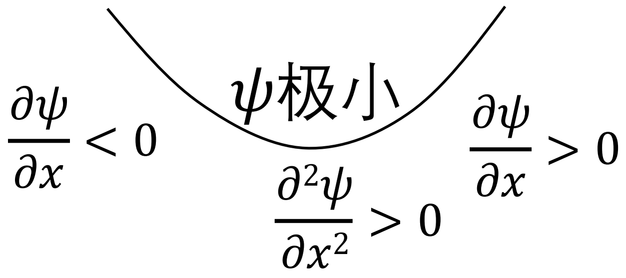
（3）全可压缩模式中，分析不同的影响因子或者背景与风暴尺度各自的影响时，一般仅计算其中两项，用余差求最后项。

(3) 动力和浮力扰动气压

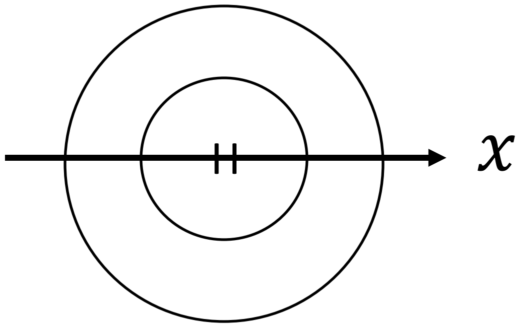
b. 物理解释

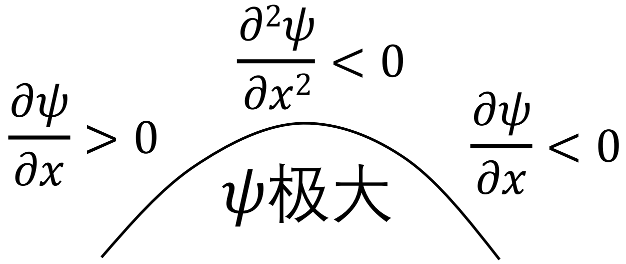
拉普拉斯算子的特征 $\nabla^2\psi = \dfrac{\partial^2\psi}{\partial x^2} + \dfrac{\partial^2\psi}{\partial y^2} + \dfrac{\partial^2\psi}{\partial z^2}$





$\Rightarrow$ 正的$\nabla^2 p'$ $\propto$ 负的p' $\Rightarrow$ $\nabla^2 p' \propto -p'$



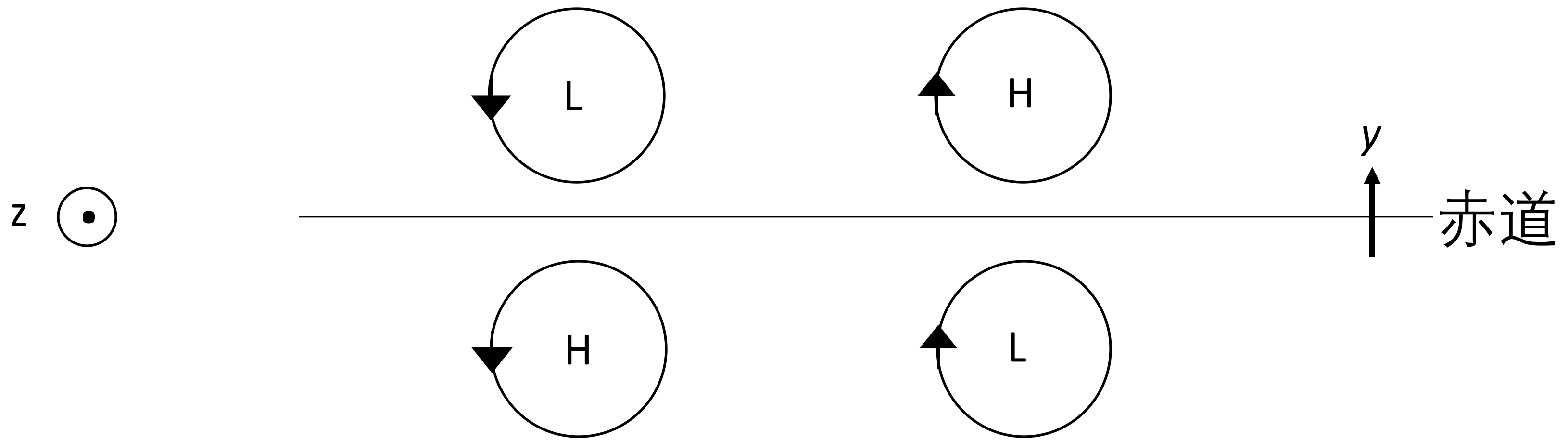


(3) 动力和浮力扰动气压

$$\nabla^2 p' = -\rho_0 \nabla \cdot (\vec{\boldsymbol{v}} \cdot \nabla \vec{\boldsymbol{v}}) + \rho_0 \frac{\partial B}{\partial z} - \rho_0 \mathrm{f}\, \nabla \cdot (\vec{\mathrm{k}} \times \vec{\boldsymbol{v}})$$

$\nabla^2 p_{\mathrm{D}}'$	$\nabla^2 p_{\mathrm{B}}'$	$\nabla^2 p_{\mathrm{G}}'$
动力	浮力	地转

1) 地转部分（大尺度） $\nabla^2 p_{\mathrm{G}}' = \rho_0 \mathrm{f}\, \zeta \Rightarrow p_{\mathrm{G}}' \propto -\rho_0 \mathrm{f}\, \zeta$



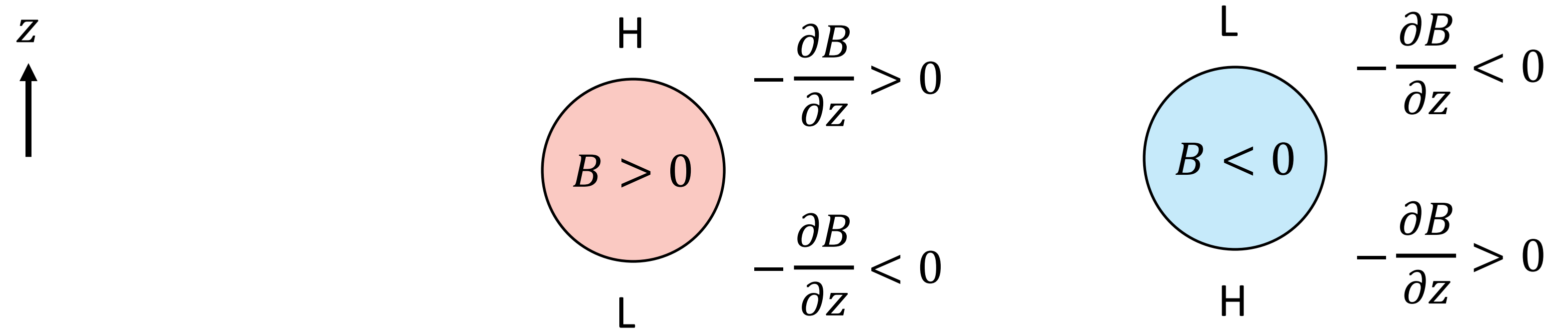
(3) 动力和浮力扰动气压

$$\nabla^2 p' = -\rho_0 \nabla \cdot (\vec{v} \cdot \nabla \vec{v}) + \rho_0 \frac{\partial B}{\partial z} - \rho_0 f \nabla \cdot (\vec{k} \times \vec{v})$$

$\nabla^2 p_D'$	$\nabla^2 p_B'$	$\nabla^2 p_G'$
动力	浮力	地转

2）浮力部分 $\nabla^2 p_B' = \rho_0 \frac{\partial B}{\partial z} \Rightarrow p_B' \propto -\frac{\partial B}{\partial z}$

$$B = -\frac{\rho'}{\rho_0} g$$



密度扰动引起的浮力对重力加速度的贡献与浮力扰动气压梯度力方向相反，在静力平衡下二者抵消

(3) 动力和浮力扰动气压

3）动力部分

$$\nabla^2 p_D' = -\rho_0 \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] - 2\rho_0 \left[\frac{\partial v}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial u}{\partial z} + \frac{\partial w}{\partial y}\frac{\partial v}{\partial z}\right]$$

三维涡度（spin） $\vec{\omega} = \left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$

变形算子（splat） $e_{ij}^2 = \frac{1}{4}\sum_{i=1}^{3}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)^2$

$$u_1 = u; u_2 = v; u_3 = w \qquad x_1 = x; x_2 = y; x_3 = z$$

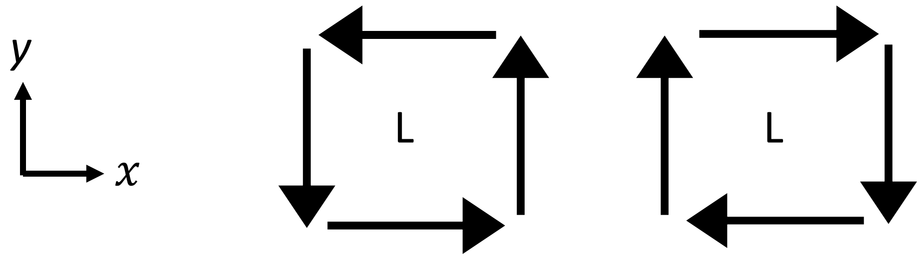
$$\nabla^2 p_D' = \rho_0 \left[\frac{1}{2}|\vec{\omega}|^2 - e_{ij}^2\right] \Rightarrow p_D' \propto e_{ij}^2 - \frac{1}{2}|\vec{\omega}|^2$$

(3) 动力和浮力扰动气压

$$p_D' \propto e_{ij}^2 - \frac{1}{2}|\vec{\omega}|^2$$

$$\nabla^2 p_D' = -\rho_0\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] - 2\rho_0\left[\frac{\partial v}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial u}{\partial z} + \frac{\partial w}{\partial y}\frac{\partial v}{\partial z}\right]$$

旋转永远伴随着扰动低压



$p_D' < 0$ $p_D' < 0$

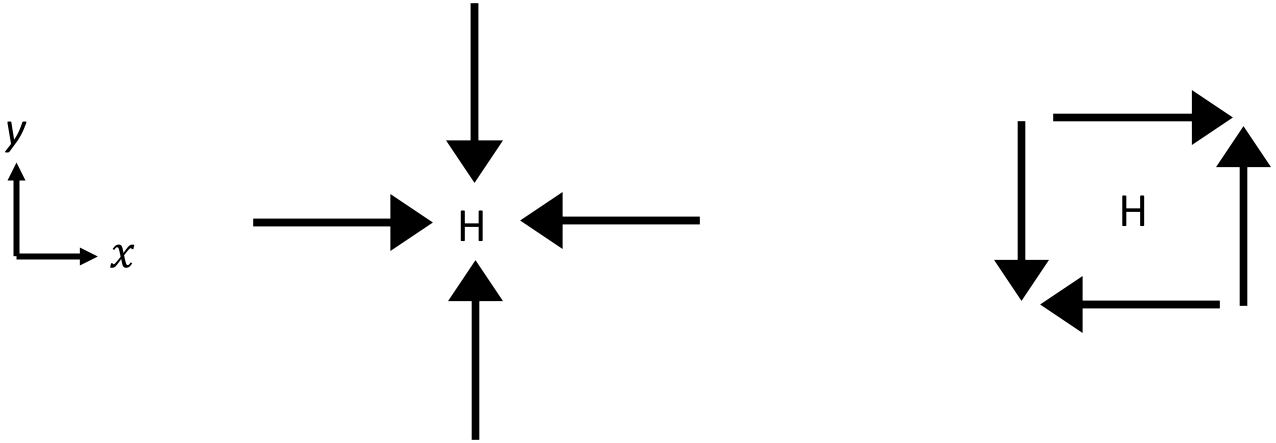
$$p_D' \propto \frac{\partial v}{\partial x}\frac{\partial u}{\partial y}$$

(3) 动力和浮力扰动气压

$$p_{\mathrm{D}}' \propto e_{ij}^2 - \frac{1}{2}|\vec{\omega}|^2$$

$$\nabla^2 p_{\mathrm{D}}' = -\rho_0\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] - 2\rho_0\left[\frac{\partial v}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial u}{\partial z} + \frac{\partial w}{\partial y}\frac{\partial v}{\partial z}\right]$$

变形永远伴随着扰动高压



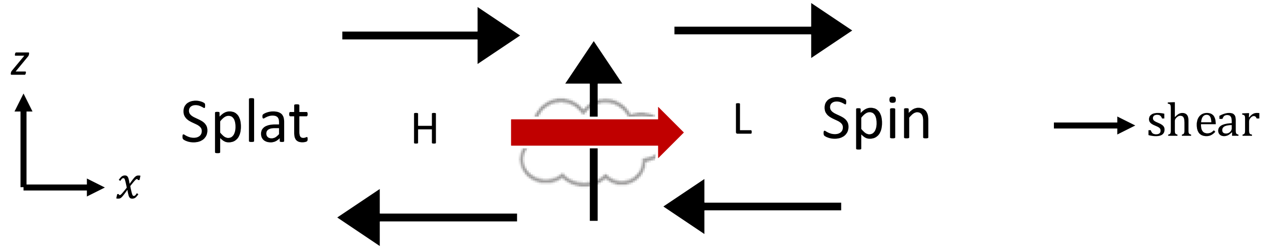
$$p_{\mathrm{D}}' \propto \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 > 0 \qquad p_{\mathrm{D}}' \propto \frac{\partial v}{\partial x}\frac{\partial u}{\partial y} > 0$$

(3) 动力和浮力扰动气压

$$p_{\mathrm{D}}' \propto e_{ij}^2 - \frac{1}{2}|\vec{\omega}|^2$$

$$\nabla^2 p_{\mathrm{D}}' = -\rho_0\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] - 2\rho_0\left[\frac{\partial v}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial u}{\partial z} + \frac{\partial w}{\partial y}\frac{\partial v}{\partial z}\right]$$

上升气块向风垂直切变的下游方向加速



$$p_{\mathrm{D}}' \propto \frac{\partial u}{\partial z}\frac{\partial w}{\partial x}$$ > 0 < 0

(4) 两种分类之间的关系

$$p' = p_{\mathrm{D}}' + p_{\mathrm{B}}'$$

$$= p_{\mathrm{D}}' + p_{\mathrm{B1}}' + p_{\mathrm{B2}}'$$

$$p' = p_{\mathrm{nh}}' + p_{\mathrm{h}}'$$

$p_{\mathrm{nh}}' \uparrow$ $p_{\mathrm{D}}' + p_{\mathrm{B1}}'$

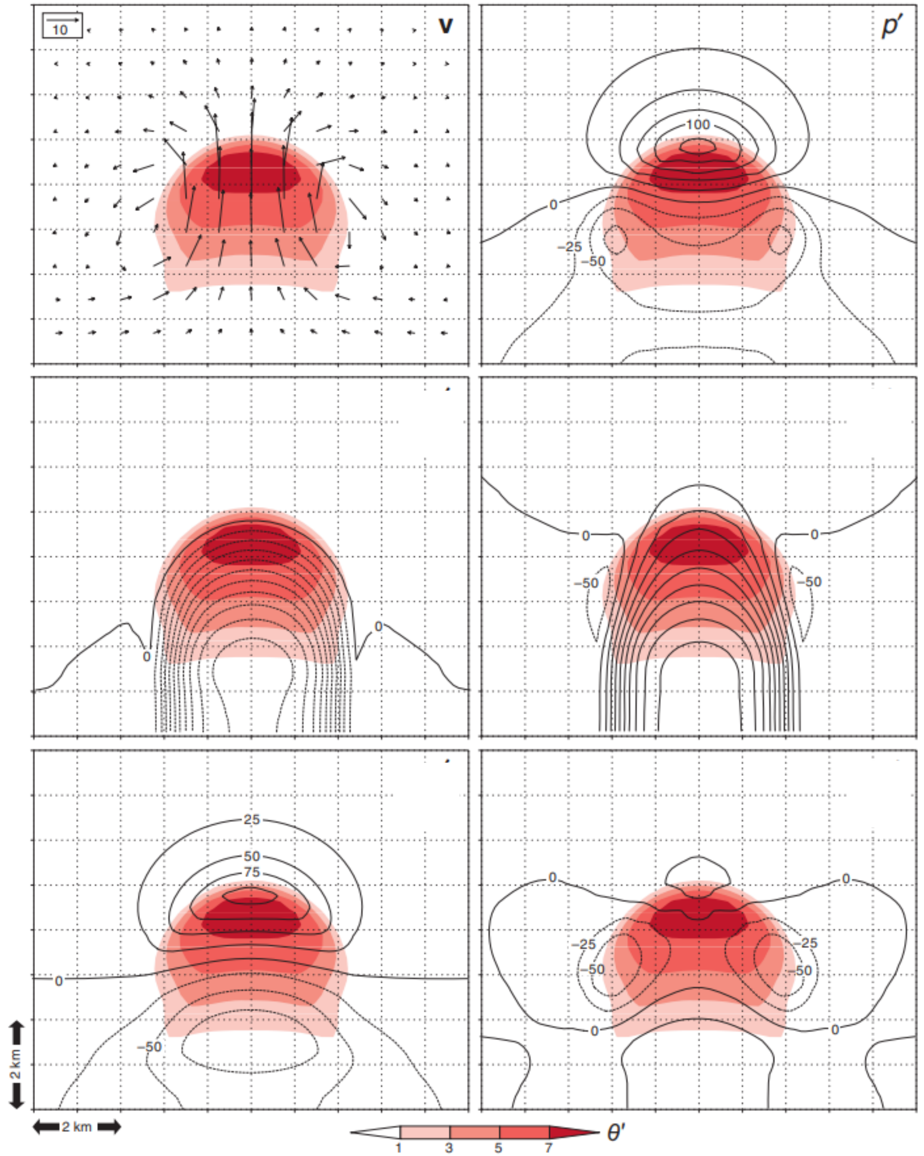
$p_{\mathrm{h}}' \uparrow$ p_{B2}'

$$\frac{\partial p'_{\mathrm{h}}}{\partial z} = -\rho' g$$

$$p_{\mathrm{B}}' \propto -\frac{\partial B}{\partial z}$$

$$p_{\mathrm{D}}' \propto e_{ij}^2 - \frac{1}{2}|\vec{\omega}|^2$$

Simulated Buoyant Updraft (grid size=100 m)
Shading: potential temperature perturbation



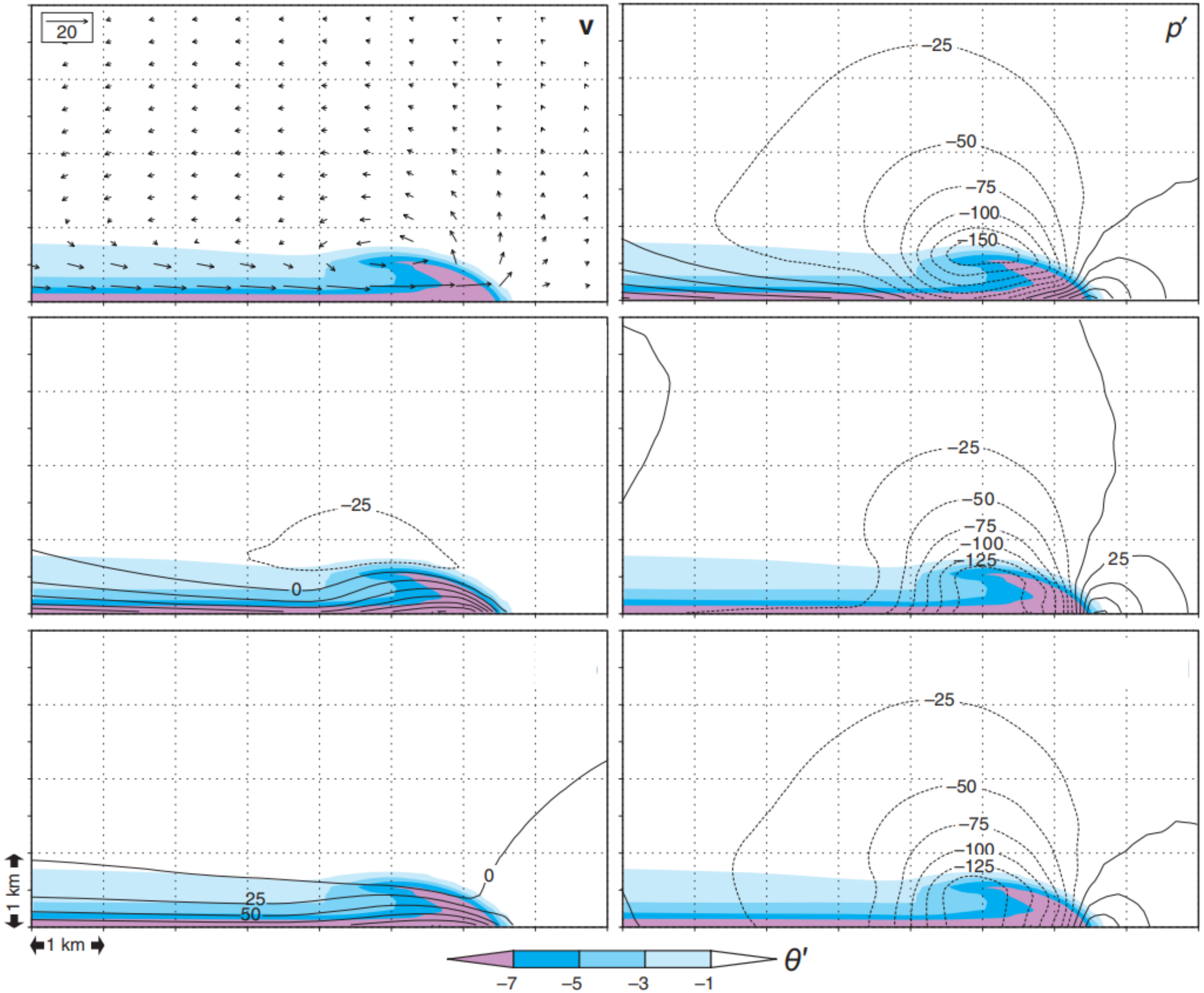
（4）两种分类之间的关系

Simulated Density current (grid size=100 m) Shading: potential temperature perturbation

$$\frac{\partial p'_{\mathrm{h}}}{\partial z} = -\rho' g$$

$$p_{\mathrm{B}}' \propto -\frac{\partial B}{\partial z}$$

$$p_{\mathrm{D}}' \propto e_{ij}^2 - \frac{1}{2}|\vec{\omega}|^2$$



(5) 线性和非线性扰动气压

$$\nabla^2 p_D{}' = -\rho_0\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] - 2\rho_0\left[\frac{\partial v}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial u}{\partial z} + \frac{\partial w}{\partial y}\frac{\partial v}{\partial z}\right]$$

$u = \bar{u} + u'$　　$v = \bar{v} + v'$　　$w = w'$

$\bar{u}, \bar{v}$ 为具有风垂直切变的背景平均气流 $\bar{u}(z)$, $\bar{v}(z)$

$$p_D{}' \propto e_{ij}^2 - \frac{1}{2}|\vec{\omega}|^2$$

$$p_D{}' \propto e_{ij}'^2 - \frac{1}{2}|\vec{\omega}'|^2 + 2\left(\frac{\partial w'}{\partial x}\frac{\partial \bar{u}}{\partial z} + \frac{\partial w'}{\partial y}\frac{\partial \bar{v}}{\partial z}\right)$$

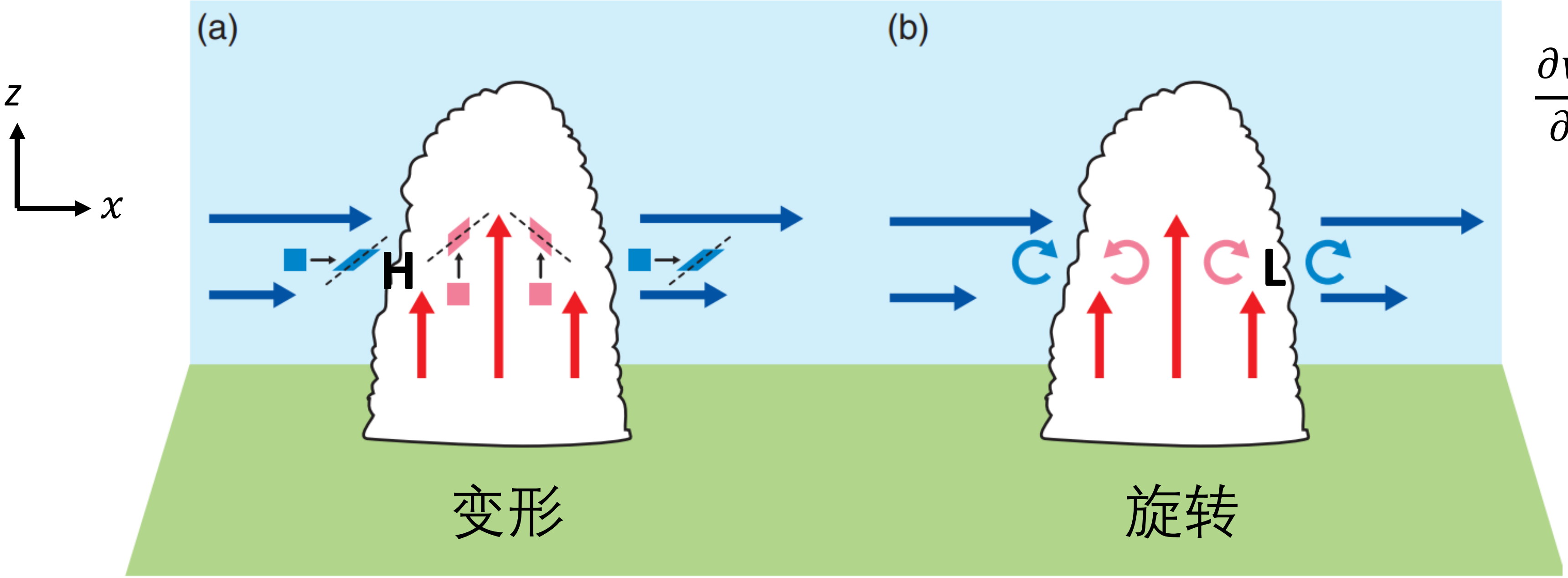
非线性动力扰动气压　　线性动力扰动气压

$a'b', (a')^2, (b')^2$　　$a'\bar{b}, b'\bar{a}, \bar{a}\bar{b}, (\bar{b})^2, (\bar{a})^2$

(5) 线性和非线性扰动气压

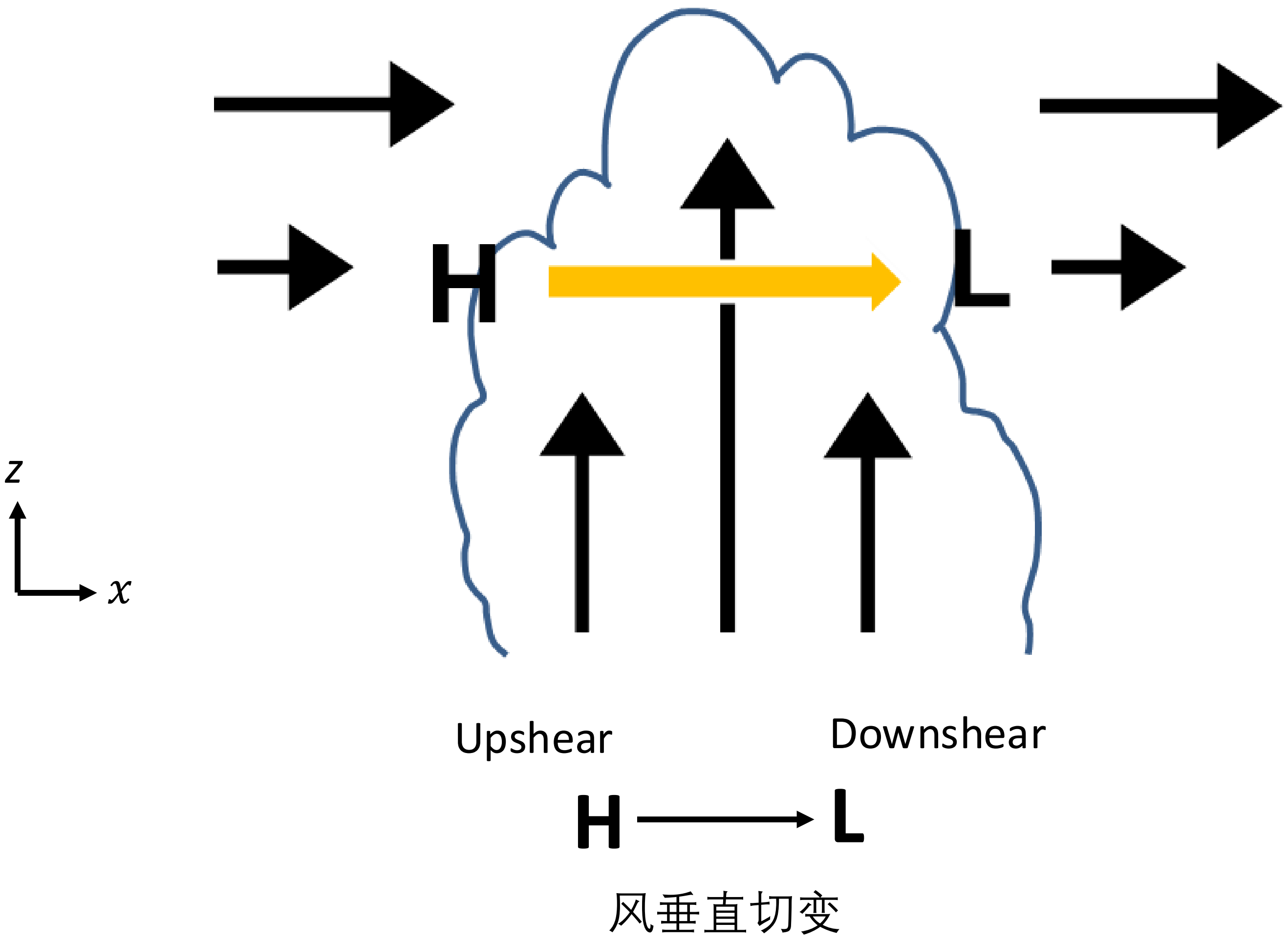
线性项 $\frac{\partial w'}{\partial x}\frac{\partial \bar{u}}{\partial z}+\frac{\partial w'}{\partial y}\frac{\partial \bar{v}}{\partial z}$

背景和扰动对变形和旋转的作用在切变上下游的共同作用可用来解释切变背景下指向切变下游方向的水平气压梯度力。



$\frac{\partial w'}{\partial x}\frac{\partial \bar{u}}{\partial z}$

(5) 线性和非线性扰动气压



可用于解释超级单体中的垂直扰动气压梯度力