Homework 1 (deadline: Oct. 10)

Demonstrate barotropic instability numerically based on the shallow water quasi-geostrophic equation in the presence of a background zonal wind shear $U(y)$. You can build your own code based on the python code shown in the class (the python code "solve_2D_nonlinear_case1.ipynb" can be downloaded from the course website `https://qiuyang50.github.io/_pages/modeling_2024fall/`). The shallow water quasi-geostrophic equation reads as follows,

$$\frac{D}{Dt}\left(\nabla^2\psi - f^2F\psi\right) + \beta\frac{\partial\psi}{\partial x} = 0$$

where $\frac{D}{Dt} = \frac{\partial}{\partial t} - \frac{\partial\psi}{\partial y}\frac{\partial}{\partial x} + \frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}$, f, F, β are all dimensionless constant, $U(y)$ has the following profile,

$$U(y) = \begin{cases} \frac{2-y}{b} & \text{if} \quad 2-b \leq y \leq 2 \\ 1 & \text{if} \quad a \leq y < 2-b \\ \frac{y}{a} & \text{if} \quad -a \leq y < a \\ -1 & \text{if} \quad b-2 \leq y < -a \\ -\frac{y+2}{b} & \text{if} \quad -2 \leq y < b-2 \end{cases}$$

where $b = 0.25$, a is a tunable parameter to determine the width of the wind shear in the middle.

Step 1: initialize the variable ψ. Hint: you can set the initial value of ψ as a combination of background state and a perturbation, $\psi(x, y, 0) = \Psi(y) + \psi'$, where $\Psi(y)$ satisfies $U(y) = -\frac{\partial\Psi(y)}{\partial y}$. You can use fast Fourier transform method (FFT) as we did in the python code to calculate $\Psi(y)$.

Step 2: add a perturbation ψ', which has to be small in amplitude and periodic at the domain boundaries. For example,

$$\psi' = 10^{-4}\sin\left(\frac{2\pi k}{L}x\right)e^{-\frac{y^2}{a^2}}$$

You may try other forms of perturbations.

Step 3: consider multiple scenarios with different values of a and k (e.g., $a = 0.5, k = 3$) and compare your results with the theoretical prediction mentioned in the class. Hint: $a*k$ should be small enough for the emergence of instability. See chapter 6.2 (page 256) in Vallis's book.

Homework requirement: please summarize all you findings in a report, particularly including several snapshots of the potential vorticity field $PV = \nabla^2\psi - f^2F\psi$ to demonstrate the barotropic instability. A thorough discussion about the choice of a and k in terms of barotropic instability is appreciated.

Tips: the whole simulation takes about 30 mins to finish. If you wish to store less output to save disk, just increase the value of the parameter "ngap" (solutions are saved every ngap steps).