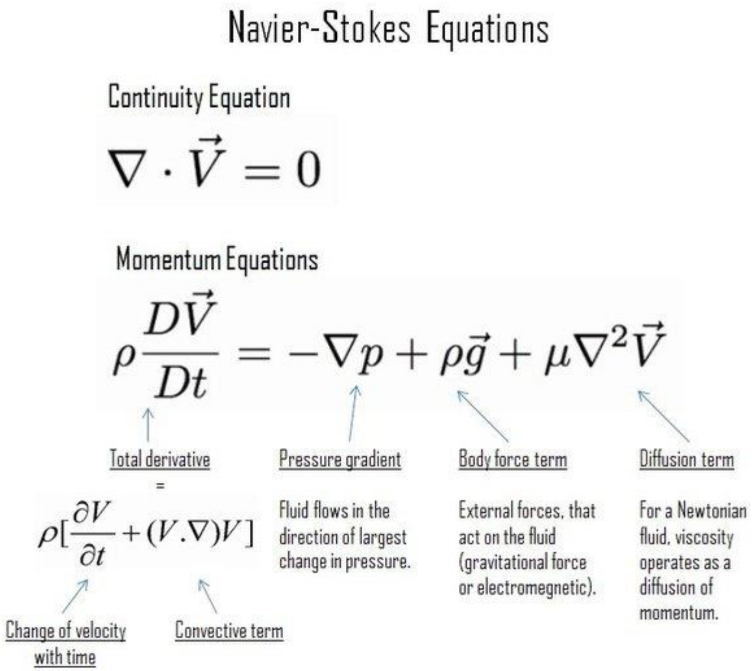
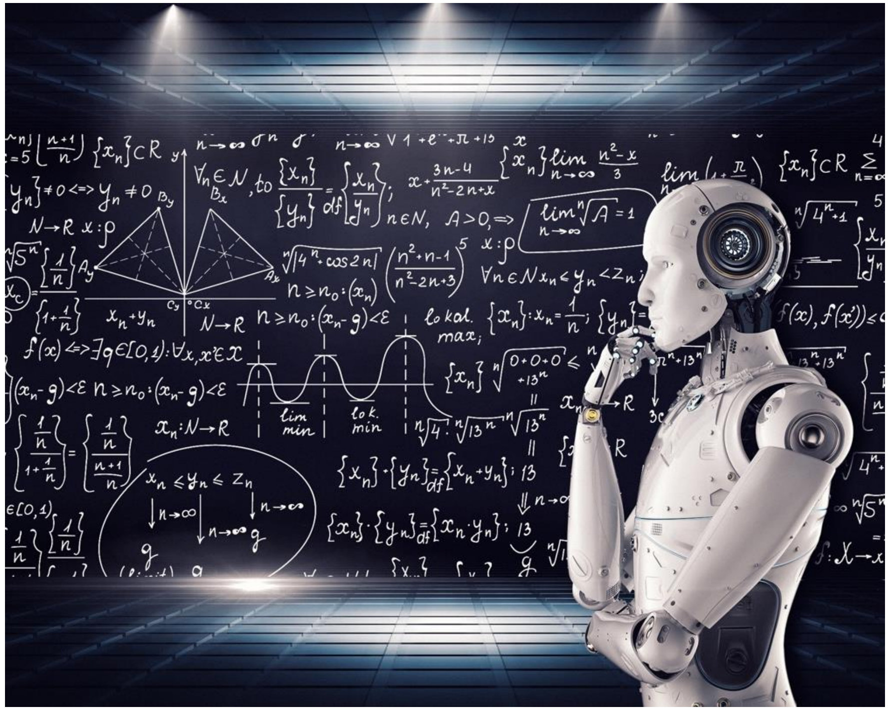
上节课回顾

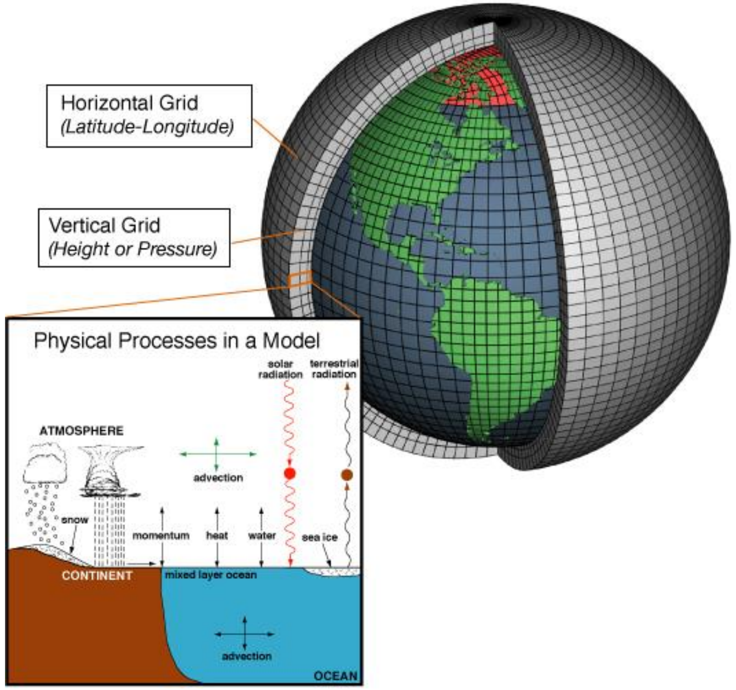
第一章



第二章



第三章



1. 请查看课程网站，上完每一章后将上传相应材料（安排、课件、作业）

   Please find the course material from the following website

   https://qiuyang50.github.io/_pages/modeling_2024fall/

地球流体的数值模拟与AI预测

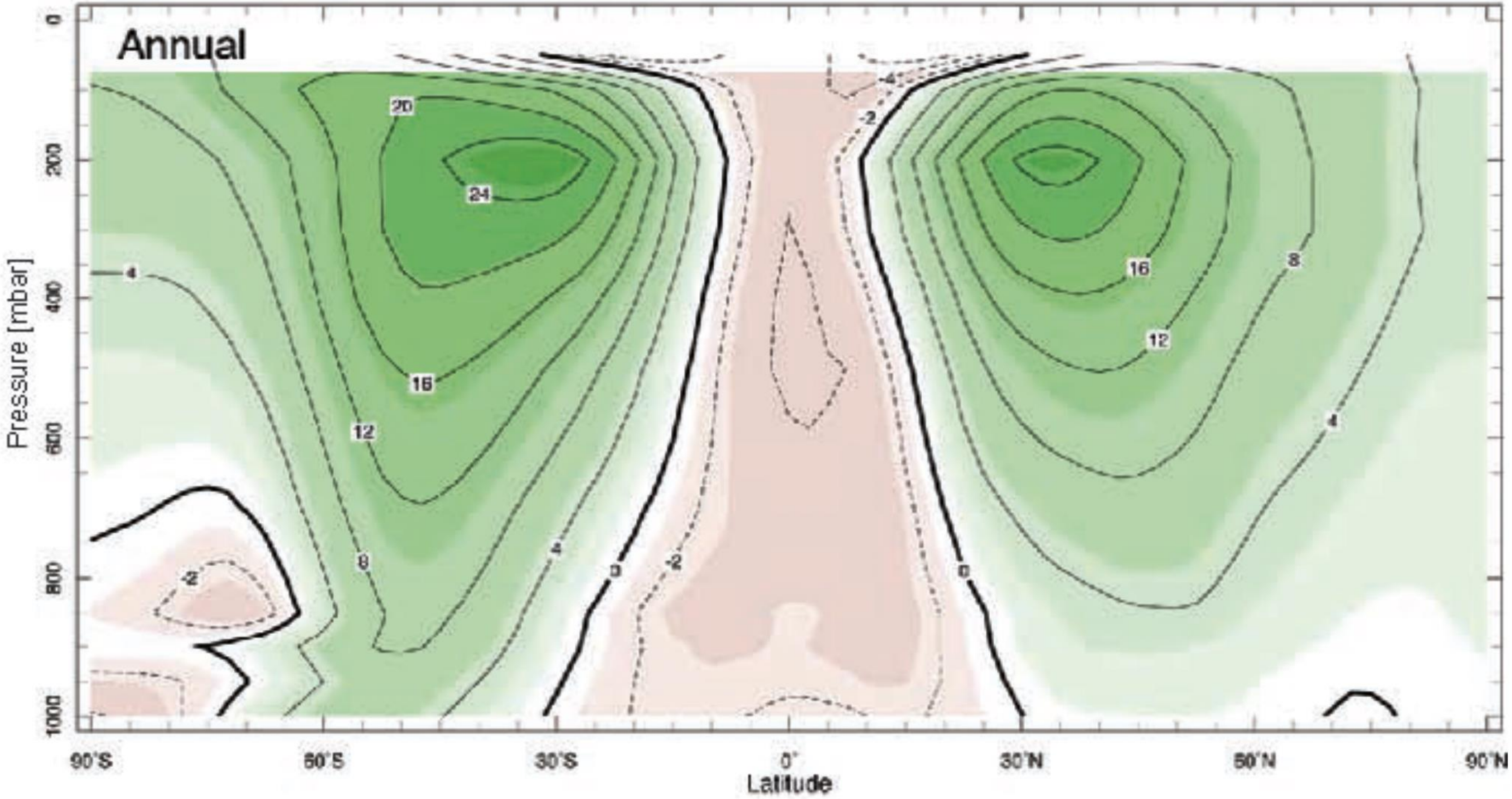
2. 请加入课程微信群（通知、交流），群内请注明真实姓名

   Please join the Wechat group for receiving future notification

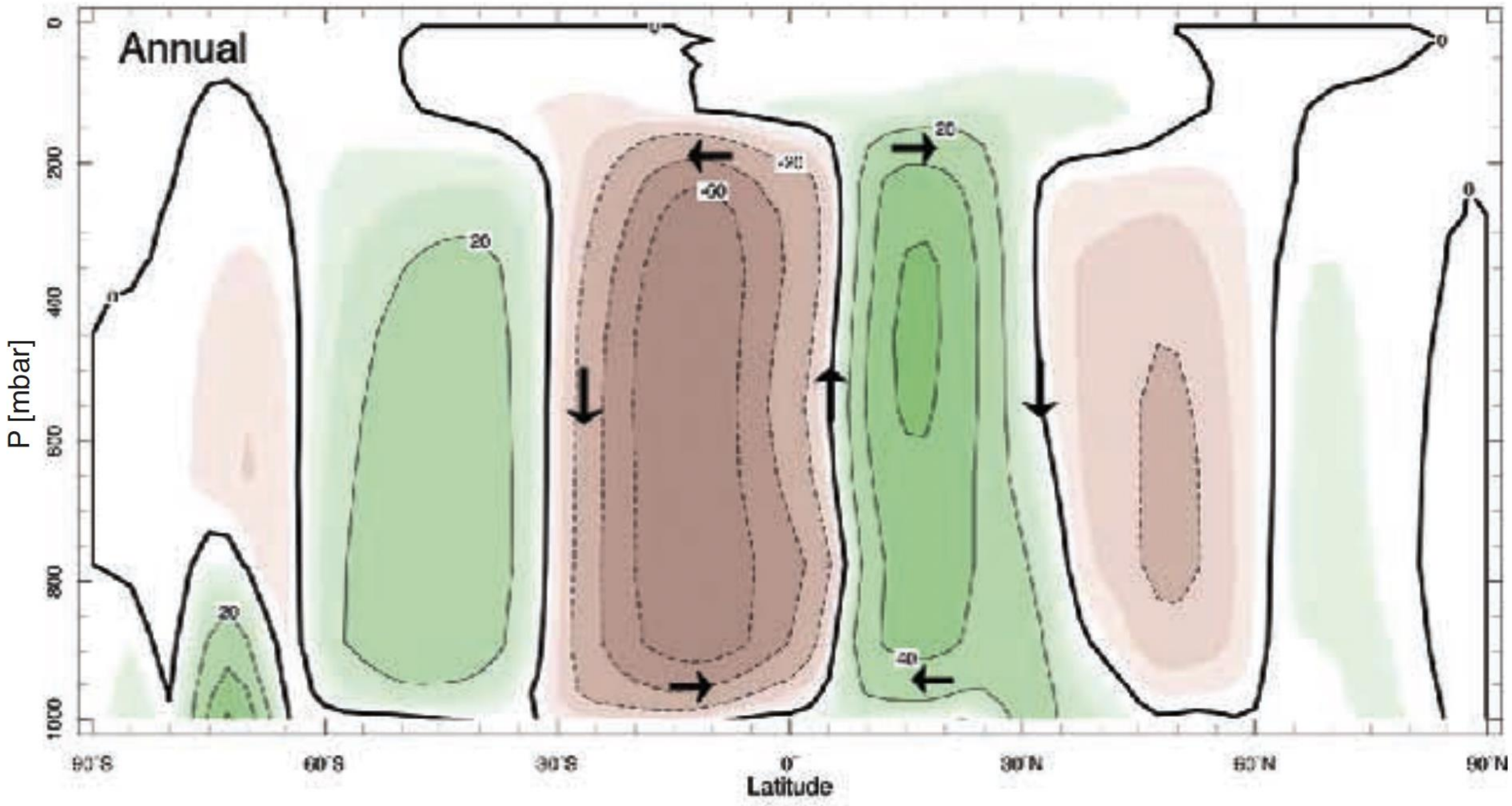
群聊: 地球流体的数值模拟和 AI 预测

1.2 基本控制方程

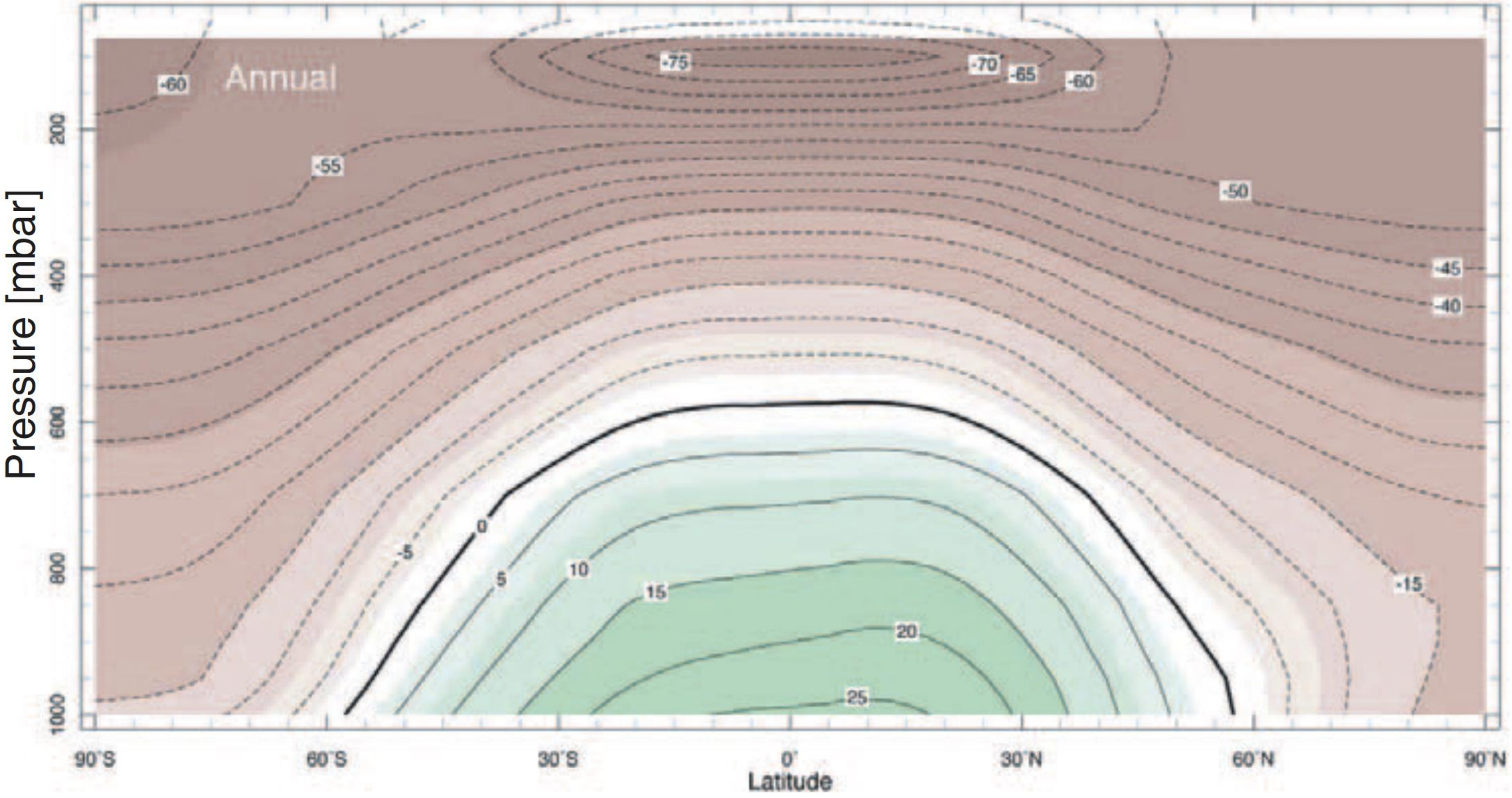
Zonal-Average, Zonal-Wind (m/s)



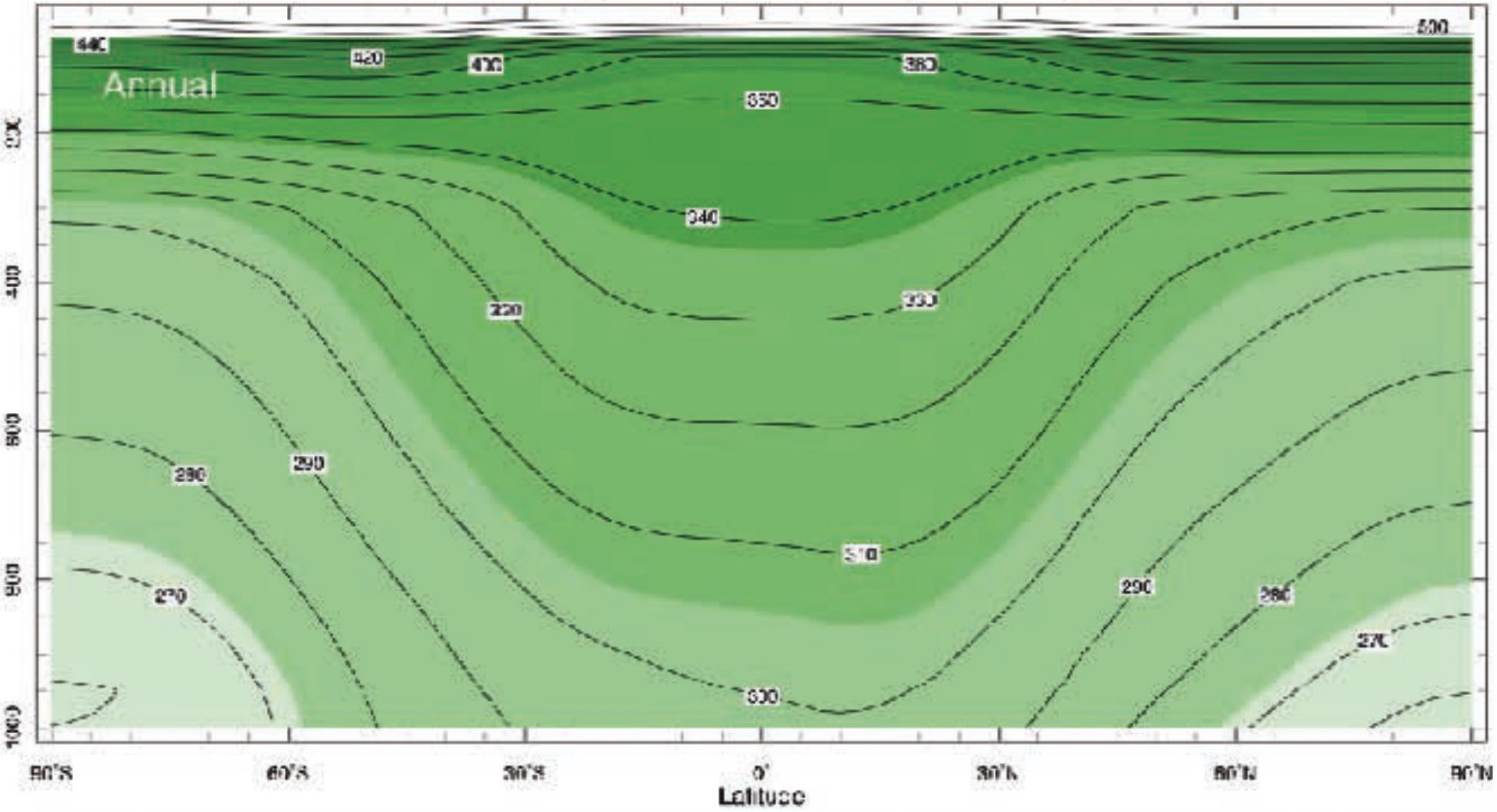
Meridional Overturning Circulation (10^9 kg/s)



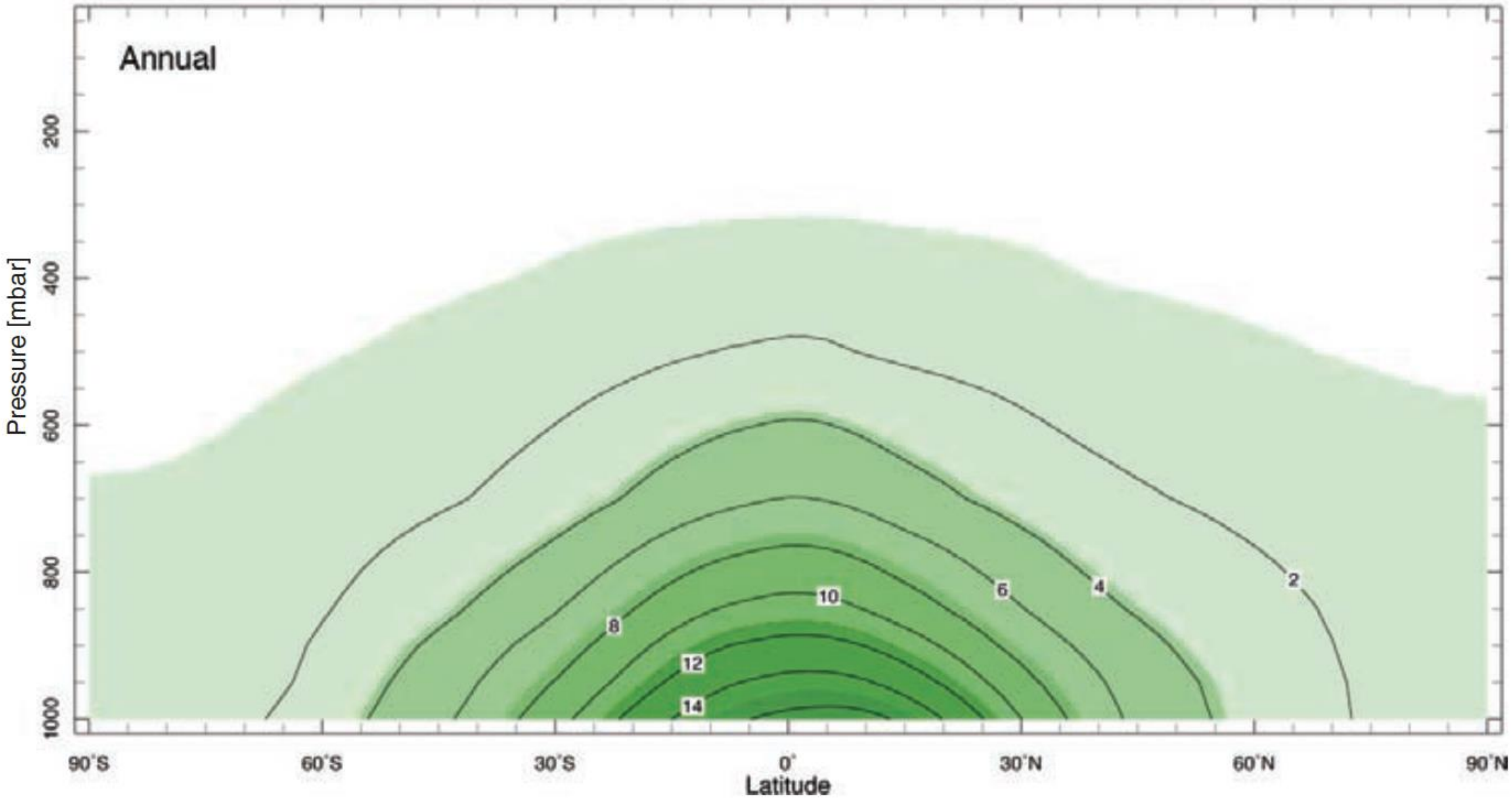
Zonal-Average Temperature (°C)



Zonal-Average Potential Temperature (K)



Zonal-Average Specific Humidity (g/kg)



Material Derivative

$$\frac{D\phi}{Dt} = \frac{\partial \phi}{\partial t} + (\boldsymbol{v} \cdot \nabla)\phi$$

Momentum Conservation

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\frac{\nabla p}{\rho} + \boldsymbol{F}',$$

Mass Conservation

$$\frac{D\rho}{Dt} + \rho\nabla \cdot \boldsymbol{v} = 0, \qquad \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0$$

The Equation of State

$$p = \rho R T,$$

Thermodynamics Equation

$$\boxed{\frac{\mathrm{D}I}{\mathrm{D}t} + p\alpha\nabla \cdot \boldsymbol{v} = \dot{Q}_T}\ .$$

$$\boxed{c_p \frac{\mathrm{D}\theta}{\mathrm{D}t} = \frac{\theta}{T}\dot{Q}}\ ,$$

The Energy Budget

$$\frac{\partial}{\partial t}\left[\rho\left(\frac{1}{2}\boldsymbol{v}^2 + I + \Phi\right)\right] + \nabla\cdot\left[\rho\boldsymbol{v}\left(\frac{1}{2}\boldsymbol{v}^2 + I + \Phi + p/\rho\right)\right] = 0.$$

$$\boxed{\frac{\partial E}{\partial t} + \nabla\cdot\left[\boldsymbol{v}(E+p)\right] = 0},$$

Extra terms for real atmosphere?

1. Coriolis force
2. Diabatic heating
3. Boundary flux
4. Dissipation

......

Sound Wave

We now consider, rather briefly, one of the most common phenomena in fluid dynamics, yet one which is in most circumstances relatively unimportant for geophysical fluid dynamics — sound waves. Their unimportance stems from the fact that the pressure disturbance produced by sound waves is a tiny fraction of the ambient pressure and too small to affect the circulation. For example, the ambient surface pressure in the atmosphere is about 10^5 Pa and variations due to large-scale weather phenomena are about 10^3 Pa or larger, whereas sound waves of 70 dB (i.e., a loud conversation) produce pressure variations of about 0.06 Pa. (1 dB $= 20\log_{10}(p/p_r)$ where $p_r = 2\times10^{-5}$Pa.)

The smallness of the disturbance produced by sound waves justifies a linearization of the equations of motion about a spatially uniform basic state that is a time-independent solution to the equations of motion. Thus, we write $\boldsymbol{v} = \boldsymbol{v}_0 + \boldsymbol{v}'$, $\rho = \rho_0 + \rho'$ (where a subscript

$$\frac{\partial^2 p'}{\partial t^2} = c_s^2 \nabla^2 p',$$

Boussinesq Equations

The simple Boussinesq equations are, for an inviscid fluid:

momentum equations: $$\frac{D\boldsymbol{v}}{Dt} + \boldsymbol{f} \times \boldsymbol{v} = -\nabla\phi + b\mathbf{k}, \tag{B.1}$$

mass conservation: $$\nabla \cdot \boldsymbol{v} = 0, \tag{B.2}$$

buoyancy equation: $$\frac{Db}{Dt} = \dot{b}. \tag{B.3}$$

Anelastic Approximation

$$\frac{\mathrm{D}\boldsymbol{v}}{\mathrm{D}t} + 2\boldsymbol{\Omega} \times \boldsymbol{v} = \mathbf{k} b_a - \nabla \phi$$

$$\frac{\mathrm{D}b_a}{\mathrm{D}t} = 0$$

$$\nabla \cdot (\tilde{\rho} \boldsymbol{v}) = 0$$

Hydrostatic Balance

$$\frac{\partial p}{\partial z} = -\rho g.$$

Moist Static Energy

Pressure coordinate

Pressure Coordinate

$$\boxed{\begin{aligned}\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} + \boldsymbol{f} \times \boldsymbol{u} = -\nabla_p \Phi, \qquad \frac{\partial \Phi}{\partial p} = -\alpha \\ \frac{\mathrm{D}\theta}{\mathrm{D}t} = 0, \qquad \nabla_p \cdot \boldsymbol{u} + \frac{\partial \omega}{\partial p} = 0\end{aligned}},$$

Geostrophic Balance

$$\boxed{fu_g \equiv -\frac{1}{\rho}\frac{\partial p}{\partial y}, \qquad fv_g \equiv \frac{1}{\rho}\frac{\partial p}{\partial x}},$$

Hydrostatic + Geostrophic = Thermal Wind

pressure coordinates. For the anelastic equations, geostrophic balance may be written

$$-fv_g = -\frac{\partial \phi}{\partial x} = -\frac{1}{a\cos\vartheta}\frac{\partial \phi}{\partial \lambda}, \qquad fu_g = -\frac{\partial \phi}{\partial y} = -\frac{1}{a}\frac{\partial \phi}{\partial \vartheta}. \tag{2.204a,b}$$

Combining these relations with hydrostatic balance, $\partial\phi/\partial z = b$, gives

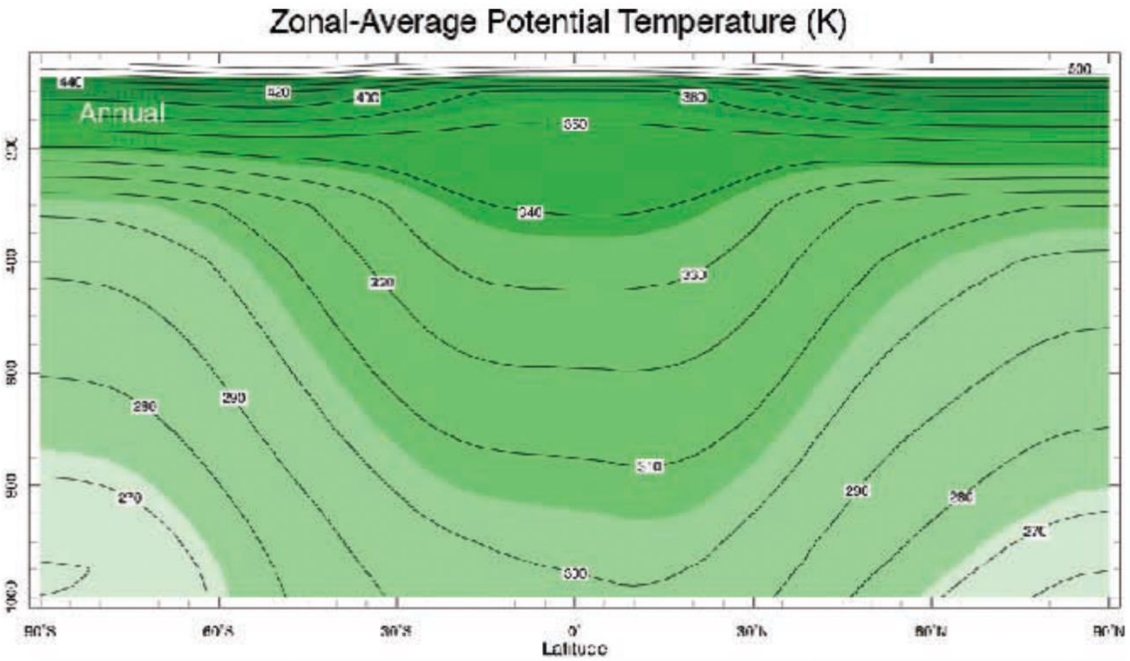
$$\boxed{\begin{aligned} -f\frac{\partial v_g}{\partial z} &= -\frac{\partial b}{\partial x} = -\frac{1}{a\cos\lambda}\frac{\partial b}{\partial \lambda} \\ f\frac{\partial u_g}{\partial z} &= -\frac{\partial b}{\partial y} = -\frac{1}{a}\frac{\partial b}{\partial \vartheta} \end{aligned}} \;. \tag{2.205a,b}$$

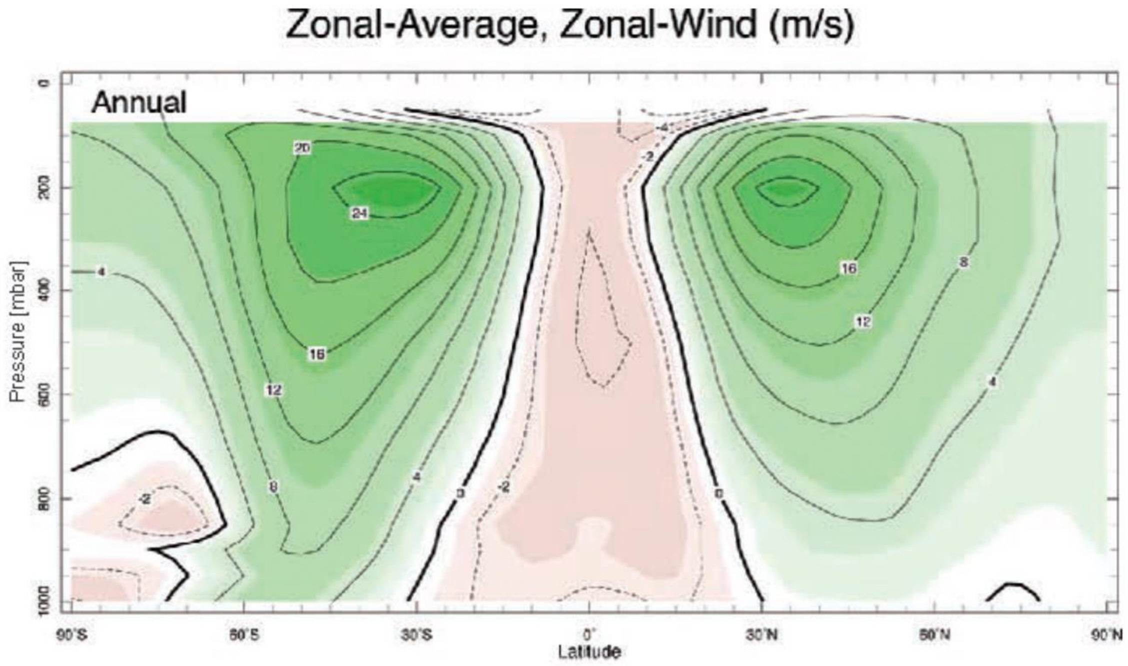
pressure coordinates. For the anelastic equations, geostrophic balance may be written

$$-fv_g = -\frac{\partial \phi}{\partial x} = -\frac{1}{a\cos\vartheta}\frac{\partial \phi}{\partial \lambda}, \qquad fu_g = -\frac{\partial \phi}{\partial y} = -\frac{1}{a}\frac{\partial \phi}{\partial \vartheta}. \tag{2.204a,b}$$

Combining these relations with hydrostatic balance, $\partial\phi/\partial z = b$, gives

$$\begin{aligned} -f\frac{\partial v_g}{\partial z} &= -\frac{\partial b}{\partial x} = -\frac{1}{a\cos\lambda}\frac{\partial b}{\partial \lambda} \\ f\frac{\partial u_g}{\partial z} &= -\frac{\partial b}{\partial y} = -\frac{1}{a}\frac{\partial b}{\partial \vartheta} \end{aligned}. \tag{2.205a,b}$$





Lapse rates in dry and moist atmospheres

$$\frac{\partial T}{\partial z} = \frac{T}{\theta}\frac{\partial \theta}{\partial z} - \frac{g}{c_p},$$

Dry

$$\begin{array}{llll} \text{stability}: & \dfrac{\partial \tilde{\theta}}{\partial z} > 0; & \text{or} & -\dfrac{\partial \tilde{T}}{\partial z} < \Gamma_d \\ \text{instability}: & \dfrac{\partial \tilde{\theta}}{\partial z} < 0; & \text{or} & -\dfrac{\partial \tilde{T}}{\partial z} > \Gamma_d \end{array},$$

is given by

$$\Gamma_d = \frac{g}{c_p},$$

Moist

$$\Gamma_s = -\frac{\mathrm{d}T}{\mathrm{d}z} = \frac{g}{c_p}\frac{1 - \rho L_c(\partial w_s/\partial p)_T}{1 + (L_c/c_p)(\partial w_s/\partial T)_p} \approx \frac{g}{c_p}\frac{1 + L_c w_s/(RT)}{1 + L_c^2 w_s/(c_p R T^2)}.$$

Gravity Waves

Let us consider a Boussinesq fluid, initially at rest, in which the buoyancy varies linearly with height and the buoyancy frequency, N, is a constant. Linearizing the equations of motion about this basic state gives the linear momentum equations,

$$\frac{\partial u'}{\partial t} = -\frac{\partial \phi'}{\partial x}, \qquad \frac{\partial w'}{\partial t} = -\frac{\partial \phi'}{\partial z} + b', \tag{2.245a,b}$$

the mass continuity and thermodynamic equations,

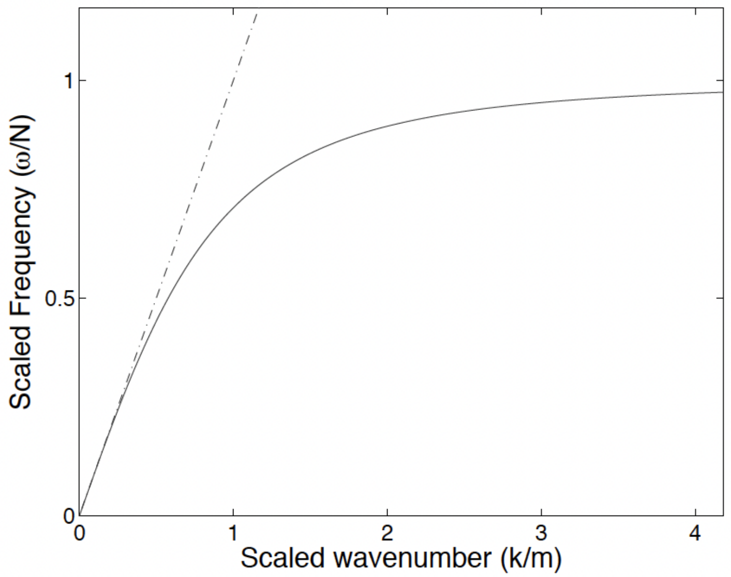
$$\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0, \qquad \frac{\partial b'}{\partial t} + w'N^2 = 0, \tag{2.246a,b}$$

where for simplicity we assume that the flow is a function only of x and z. A little algebra gives a single equation for w',

$$\left[\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\frac{\partial^2}{\partial t^2} + N^2\frac{\partial^2}{\partial x^2}\right]w' = 0. \tag{2.247}$$

Seeking solutions of the form $w' = \mathrm{Re}\, W \exp[\mathrm{i}(kx + mz - \omega t)]$ (where Re denotes the real part) yields the dispersion relationship for gravity waves:

$$\boxed{\omega^2 = \frac{k^2N^2}{k^2 + m^2}}. \tag{2.248}$$



2.2 Flux-Form Euler Equations

Using the variables defined above, the flux-form Euler equations can be written as

$$\partial_t U + (\nabla \cdot \mathbf{V} u) + \mu_d \alpha \partial_x p + (\alpha/\alpha_d) \partial_\eta p \partial_x \phi = F_U \tag{2.8}$$

$$\partial_t V + (\nabla \cdot \mathbf{V} v) + \mu_d \alpha \partial_y p + (\alpha/\alpha_d) \partial_\eta p \partial_y \phi = F_V \tag{2.9}$$

$$\partial_t W + (\nabla \cdot \mathbf{V} w) - g[(\alpha/\alpha_d) \partial_\eta p - \mu_d] = F_W \tag{2.10}$$

$$\partial_t \Theta_m + (\nabla \cdot \mathbf{V} \theta_m) = F_{\Theta_m} \tag{2.11}$$

$$\partial_t \mu_d + (\nabla \cdot \mathbf{V}) = 0 \tag{2.12}$$

$$\partial_t \phi + \mu_d^{-1}[(\mathbf{V} \cdot \nabla \phi) - gW] = 0 \tag{2.13}$$

$$\partial_t Q_m + (\nabla \cdot \mathbf{V} q_m) = F_{Q_m} \tag{2.14}$$

Numerical Scheme

$$\frac{\partial \psi}{\partial t} + c\frac{\partial \psi}{\partial x} = 0$$

$$\frac{df}{dx}(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x},$$

$$\frac{df}{dx}(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0) - f(x_0 - \Delta x)}{\Delta x},$$

$$\frac{df}{dx}(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0 - \Delta x)}{2\Delta x}.$$

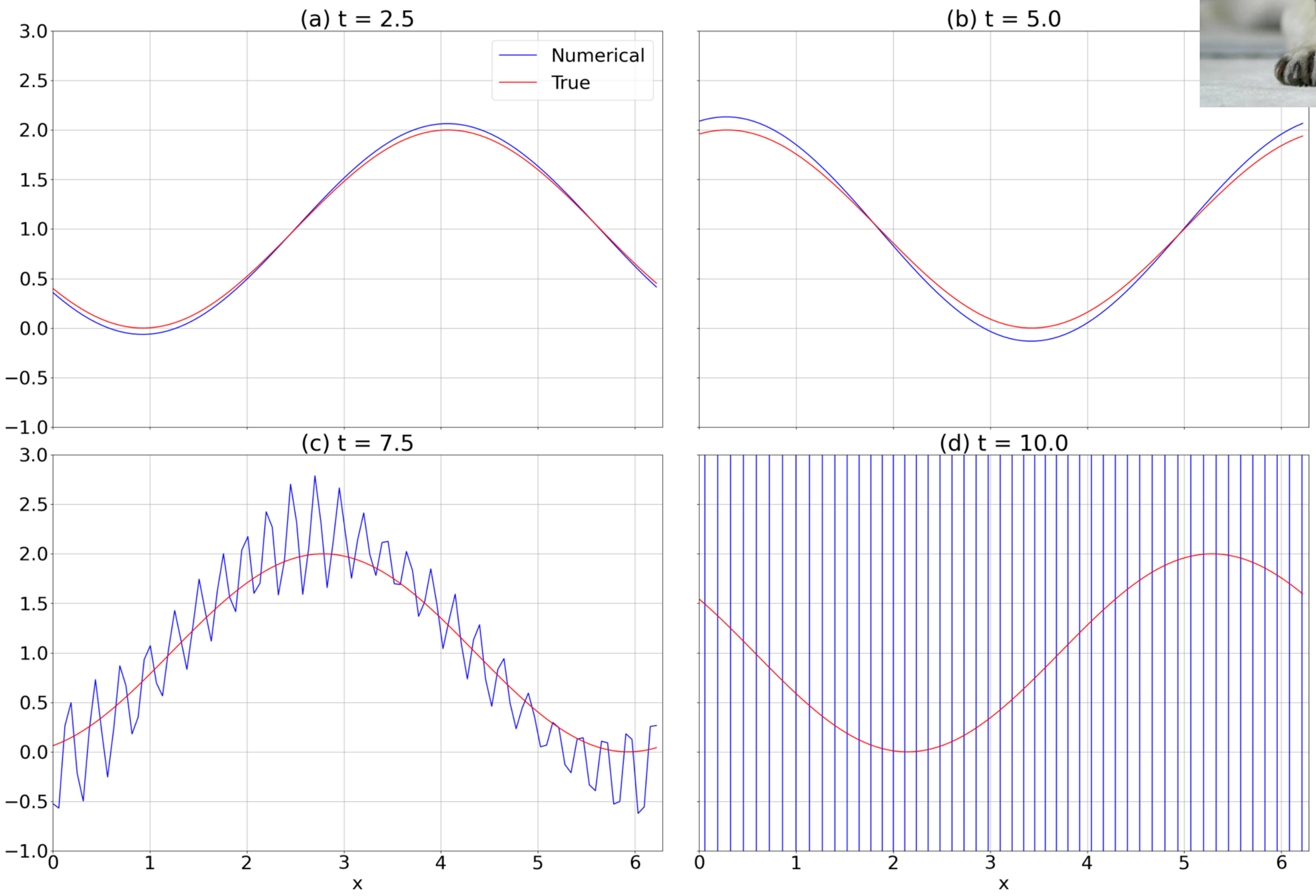
See Dale R. Durran (1998): Numerical Methods for Wave Equations in Geophysical Fluid Dynamics

参见jupyterlab python代码具体演示

建议通过安装anaconda
https://www.anaconda.com/ 来使用python编程

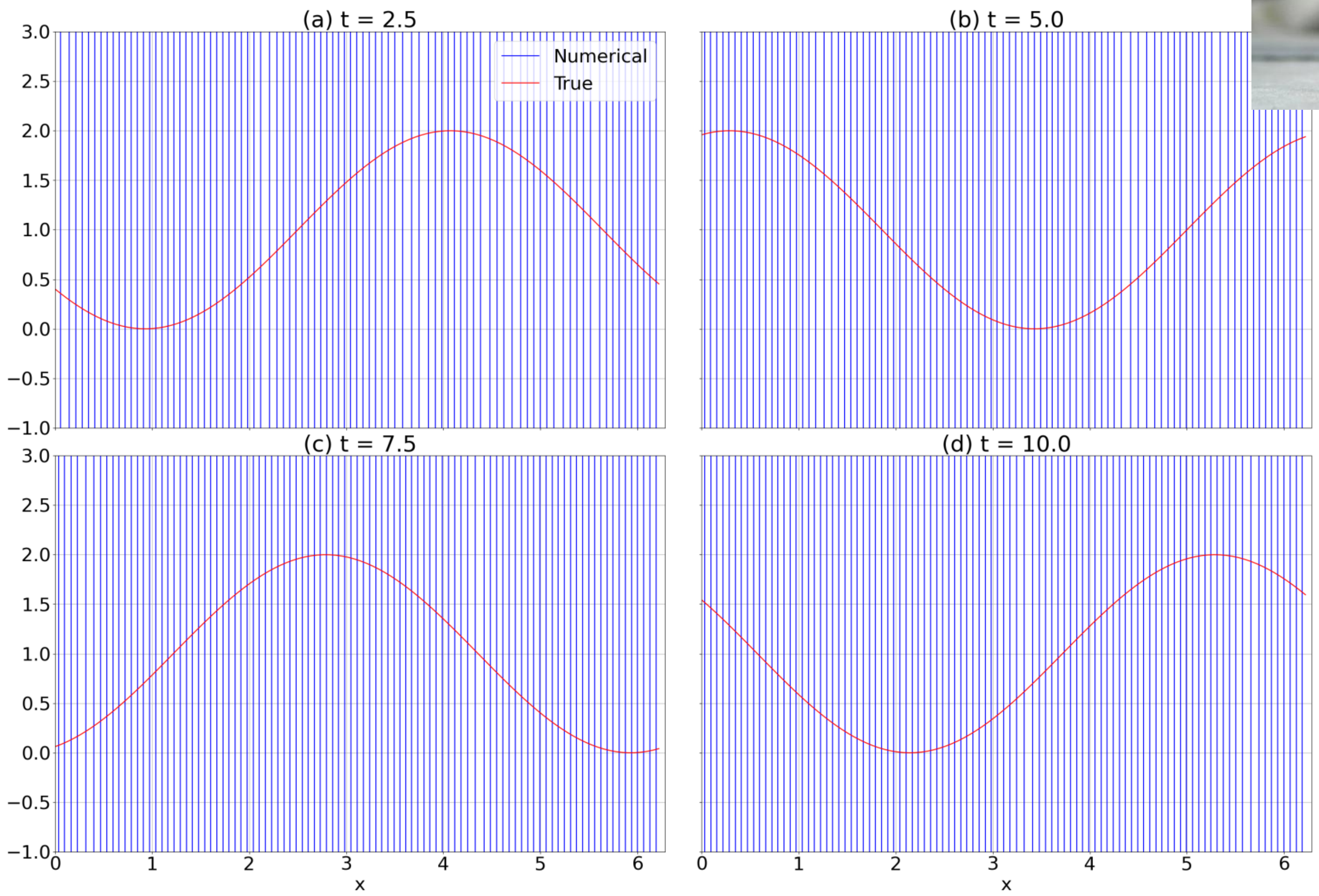
Central Difference Scheme

$$\frac{u_i^{n+1} - u_i^n}{dt} + c\,\frac{u_{i+1}^n - u_{i-1}^n}{2dx} = 0$$



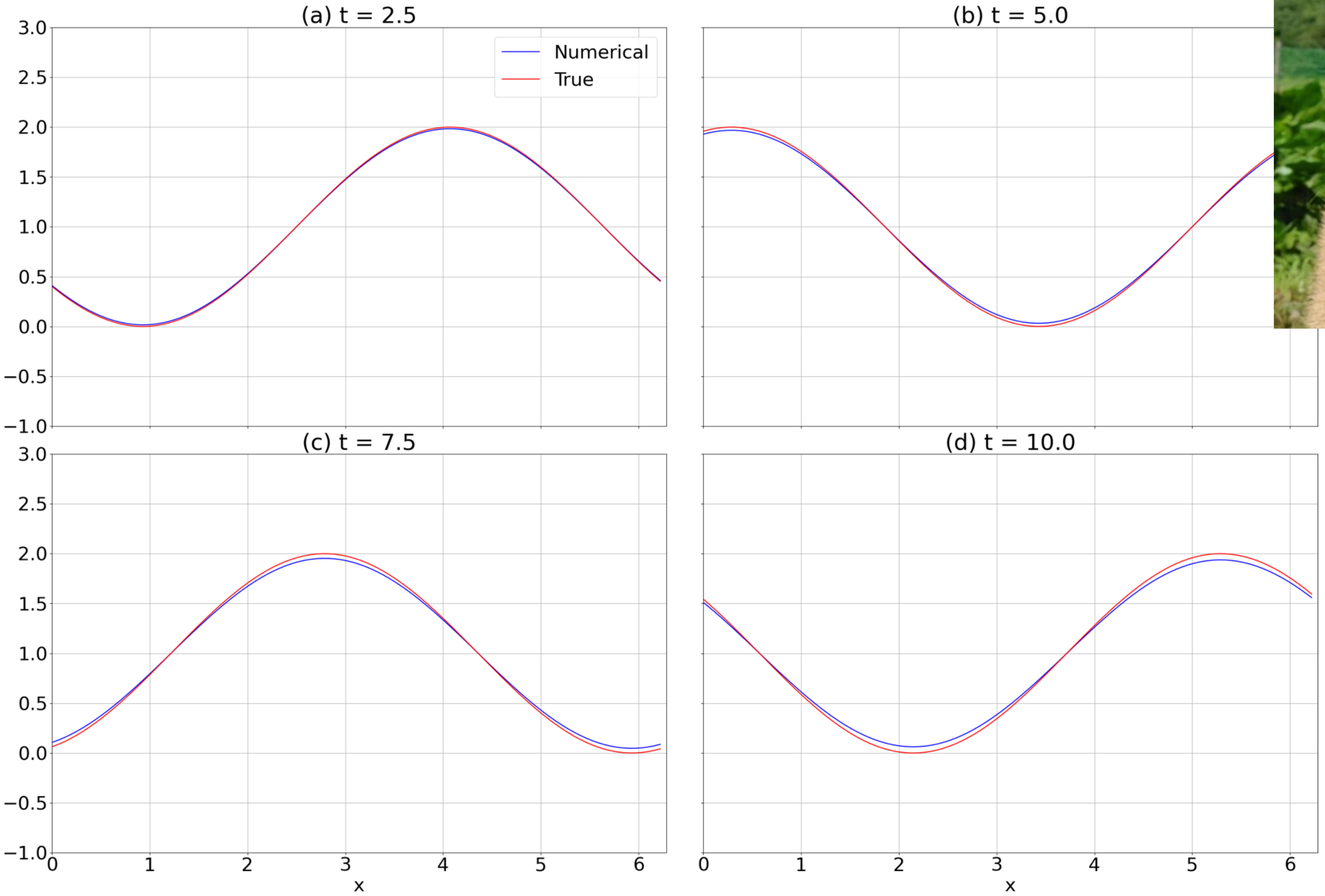
Downwind Scheme

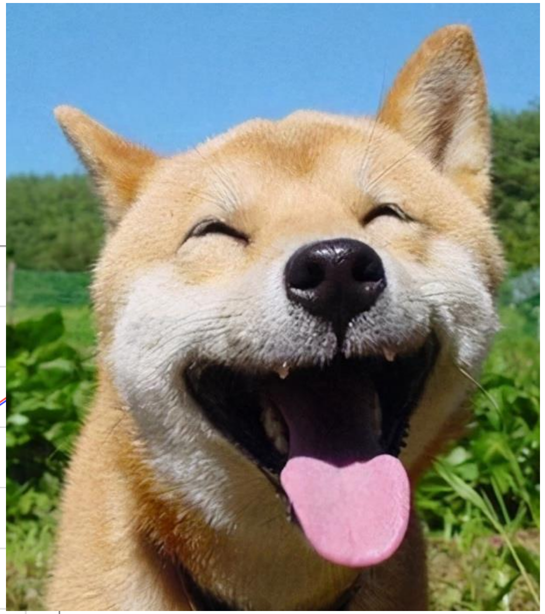
$$\frac{u_i^{n+1} - u_i^n}{dt} + c\,\frac{u_{i+1}^n - u_i^n}{dx} = 0$$



Upwind Scheme

$$\frac{u_i^{n+1} - u_i^n}{dt} + c\,\frac{u_i^n - u_{i-1}^n}{dx} = 0$$





Von Neumann’s method for proving the stability of numerical scheme

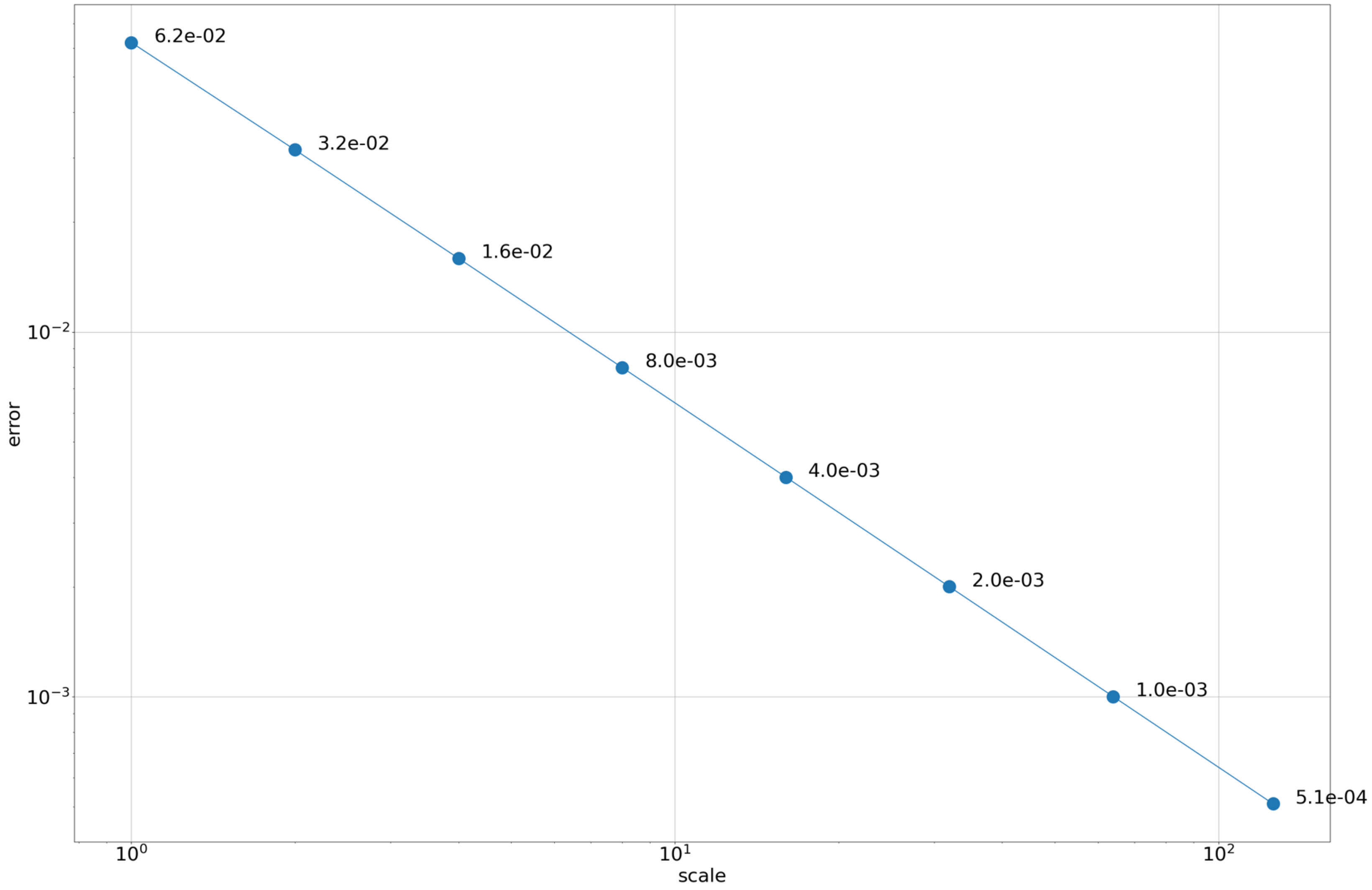
If we only reduce time step dt

dt	error
0.2	2e20
0.1	4e17
0.05	0.06
0.025	0.17

Courant-Fredrichs-Lewy (CFL) Condition

The CFL condition requires that the numerical domain of dependence of a finite difference scheme include the domain of dependence of the associated partial differential equation

Reduce dx and dt by scale times



Lax equivalence theorem
If a finite-difference scheme is linear, stable, and accurate of order (p,q), then it is convergent of order (p,q) (Lax and Richtmyer 1956).

consistency + stability = convergence