上节课回顾

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\frac{\nabla p}{\rho} + \boldsymbol{F}',$$

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} + \rho\nabla \cdot \boldsymbol{v} = 0, \qquad \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0$$

$$p = \rho R T,$$

$$\frac{\mathrm{D}I}{\mathrm{D}t} + p\alpha\nabla \cdot \boldsymbol{v} = \dot{Q}_T$$ 或 $$c_p \frac{\mathrm{D}\theta}{\mathrm{D}t} = \frac{\theta}{T}\dot{Q},$$

$$\frac{\partial}{\partial t}\left[\rho\left(\frac{1}{2}\boldsymbol{v}^2 + I + \Phi\right)\right] + \nabla \cdot \left[\rho\boldsymbol{v}\left(\frac{1}{2}\boldsymbol{v}^2 + I + \Phi + p/\rho\right)\right] = 0.$$

$$\frac{\partial E}{\partial t} + \nabla \cdot [\boldsymbol{v}(E + p)] = 0,$$

Von Neumann's method

$$\frac{\partial \psi}{\partial t} + c\frac{\partial \psi}{\partial x} = 0$$

Courant-Fredrichs-Lewy (CFL) Condition

The CFL condition requires that the numerical domain of dependence of a finite difference scheme include the domain of dependence of the associated partial differential equation

Upwind Scheme

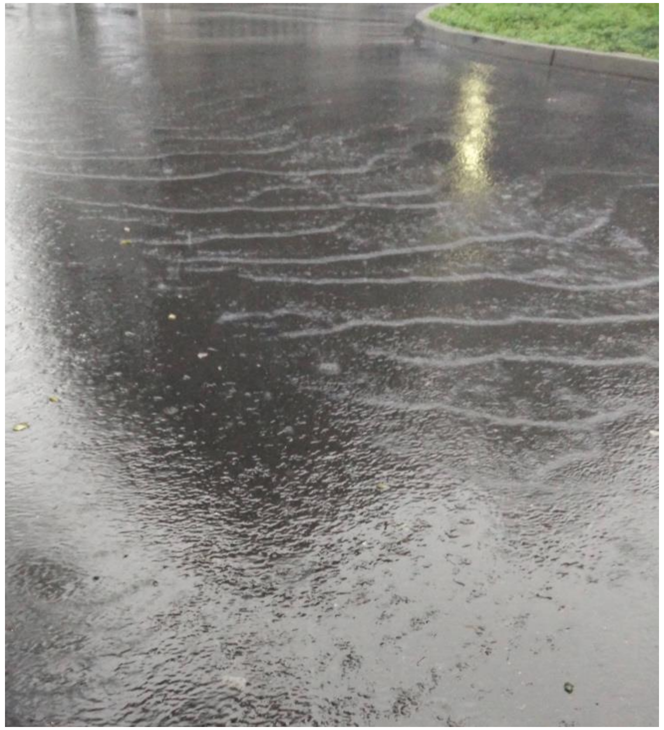
$$\frac{u_i^{n+1} - u_i^n}{dt} + c\,\frac{u_i^n - u_{i-1}^n}{dx} = 0$$

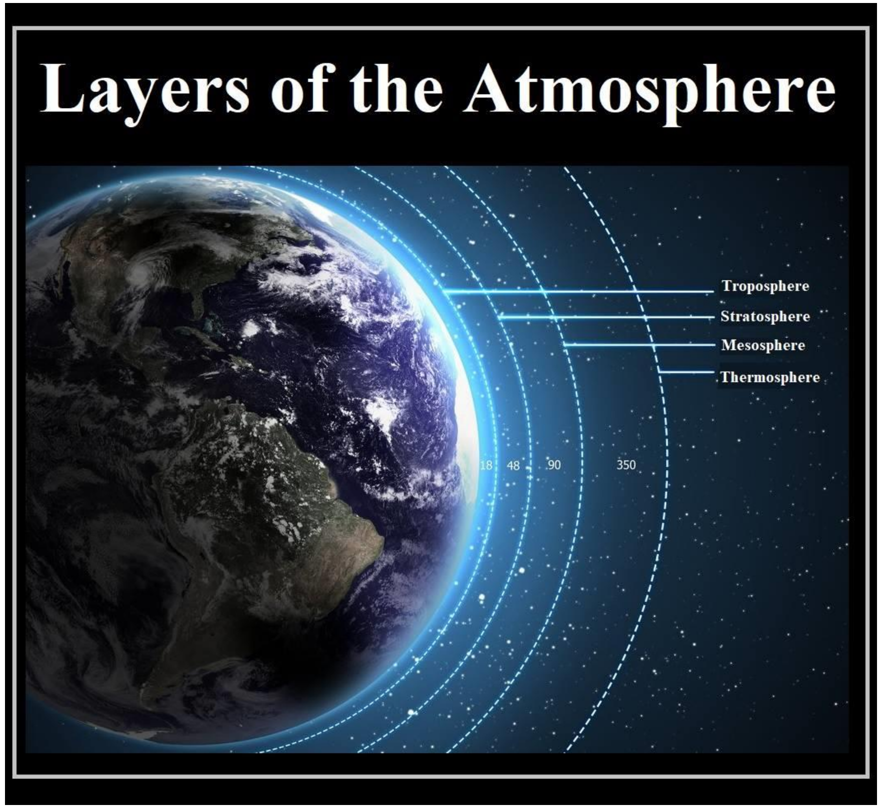
Lax equivalence theorem

If a finite-difference scheme is linear, stable, and accurate of order (p,q), then it is convergent of order (p,q) (Lax and Richtmyer 1956).

1.3 浅水模型及应用实例

Examples of Shallow Water

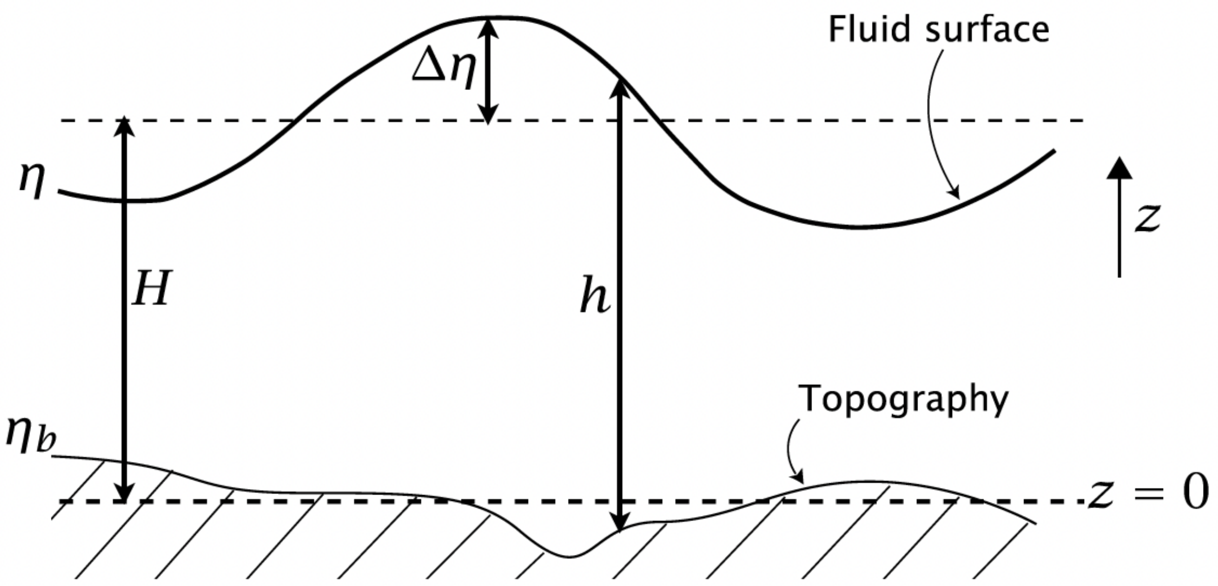


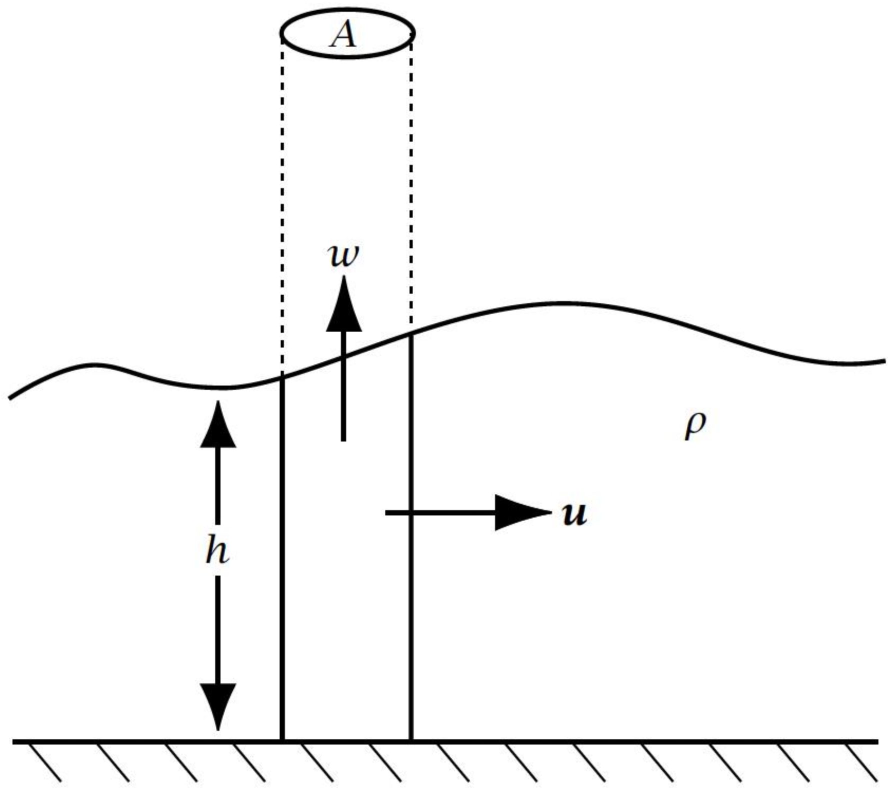


Assumption

1. Constant density
2. Small aspect ratio (vertical to horizontal)

Governing Equations

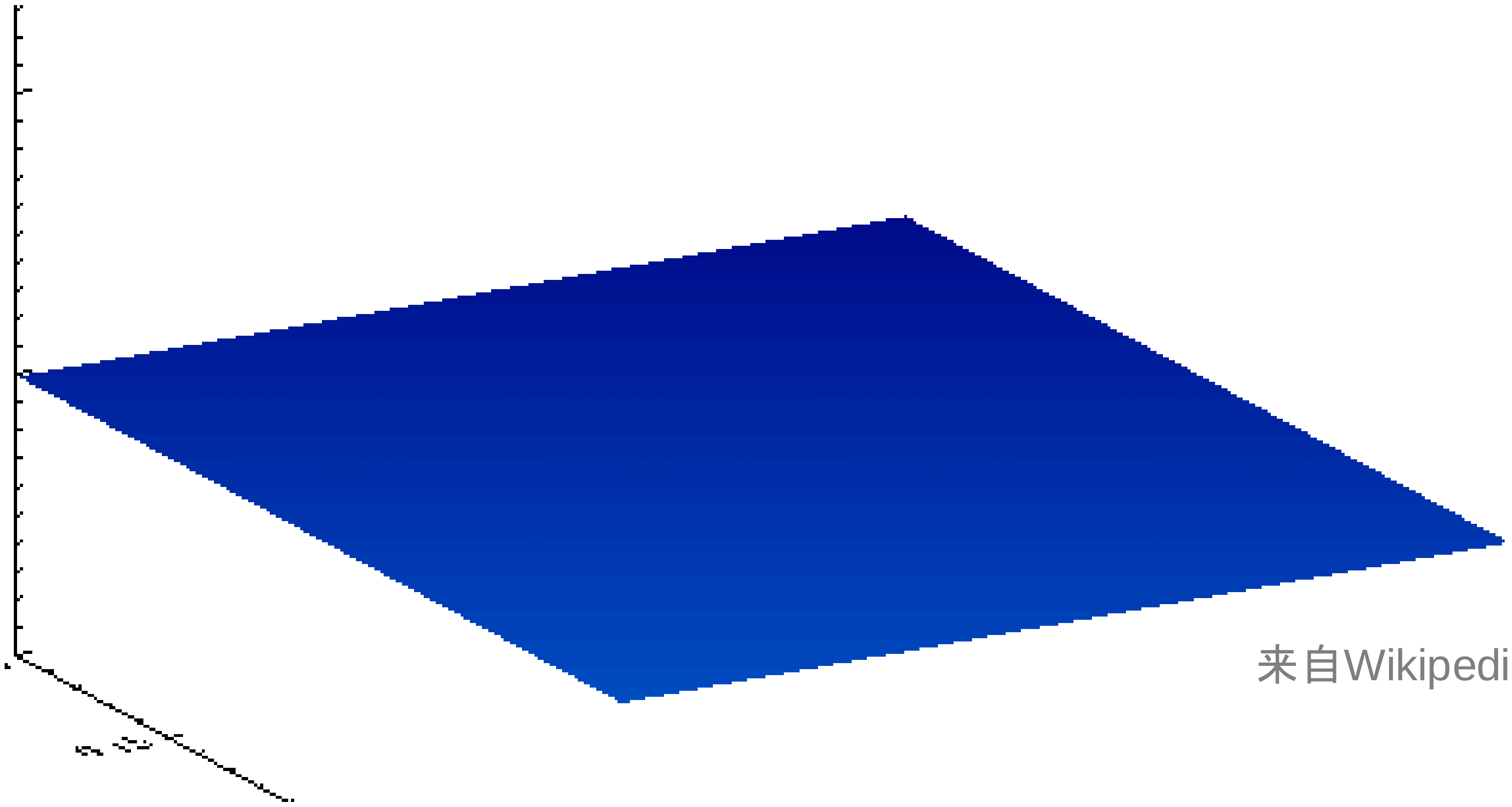




$$\boxed{\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} + \boldsymbol{f} \times \boldsymbol{u} = -g\nabla\eta},$$

$$\boxed{\frac{\mathrm{D}h}{\mathrm{D}t} + h\nabla \cdot \boldsymbol{u} = 0}.$$

$$h = \eta - \eta_b.$$



来自Wikipedia

浅水方程

$$\text{momentum:} \quad \frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} + \boldsymbol{f} \times \boldsymbol{u} = -g\nabla\eta.$$

$$\text{mass continuity:} \quad \frac{\mathrm{D}h}{\mathrm{D}t} + h\nabla \cdot \boldsymbol{u} = 0$$

Conservation Law: Energy

$$\frac{\partial}{\partial t}\frac{1}{2}\left(h\boldsymbol{u}^2 + gh^2\right) + \nabla \cdot \left[\frac{1}{2}\boldsymbol{u}\left(gh^2 + h\boldsymbol{u}^2 + gh^2\right)\right] = 0,$$

Shallow Water Waves on f-plane

$$\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} + \boldsymbol{f} \times \boldsymbol{u} = -g\nabla\eta,$$

$$\frac{\mathrm{D}h}{\mathrm{D}t} + h\nabla \cdot \boldsymbol{u} = 0.$$

$$\frac{\partial u'}{\partial t} - f_0 v' = -g\frac{\partial \eta'}{\partial x}, \qquad \frac{\partial v'}{\partial t} + f_0 u' = -g\frac{\partial \eta'}{\partial y}, \qquad \frac{\partial \eta'}{\partial t} + H\left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right) = 0. \tag{3.98a,b,c}$$

We non-dimensionalize these equations by writing

$$(x, y) = L(\hat{x}, \hat{y}), \quad (u', v') = U(\hat{u}, \hat{v}), \quad t = \frac{L}{U}\hat{t}, \quad f_0 = \frac{\hat{f}_0}{T}, \quad \eta' = H\hat{\eta}, \tag{3.99}$$

and (3.98) becomes

$$\frac{\partial \hat{u}}{\partial \hat{t}} - \hat{f}_0\hat{v} = -\hat{c}^2\frac{\partial \hat{\eta}}{\partial \hat{x}}, \qquad \frac{\partial \hat{v}}{\partial \hat{t}} + \hat{f}_0\hat{u} = -\hat{c}^2\frac{\partial \hat{\eta}}{\partial \hat{y}}, \qquad \frac{\partial \hat{\eta}}{\partial \hat{t}} + \left(\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}}\right) = 0. \tag{3.100a,b,c}$$

where $\hat{c} = \sqrt{gH}/U$ is the non-dimensional speed of non-rotating shallow water waves. (It is also the inverse of the Froude number $U/\sqrt{gH}$.) To obtain a dispersion relationship we let

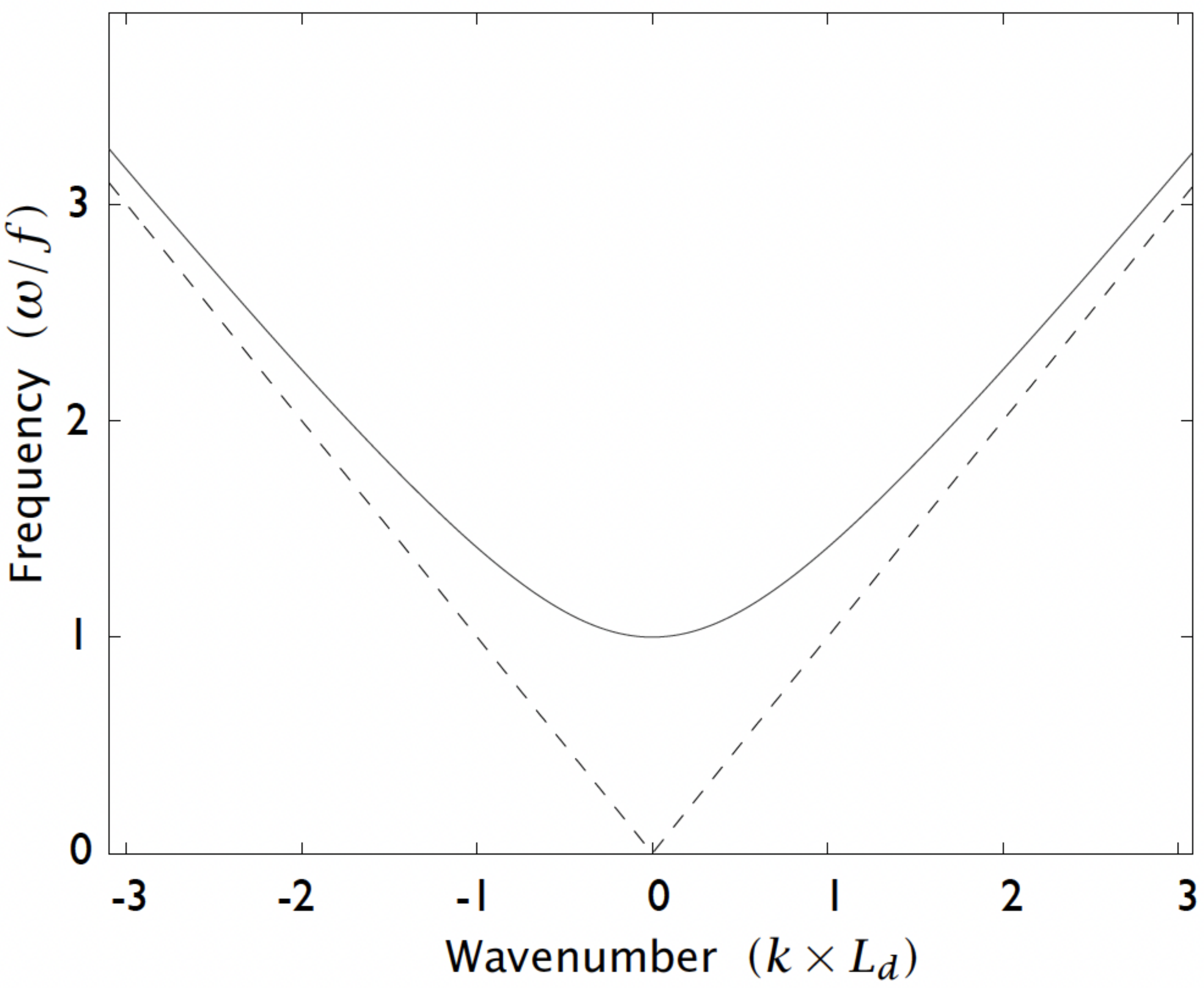
$$(\hat{u}, \hat{v}, \hat{\eta}) = (\tilde{u}, \tilde{v}, \tilde{\eta})\, \mathrm{e}^{\mathrm{i}(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{x}} - \widehat{\omega}\hat{t})}, \tag{3.101}$$

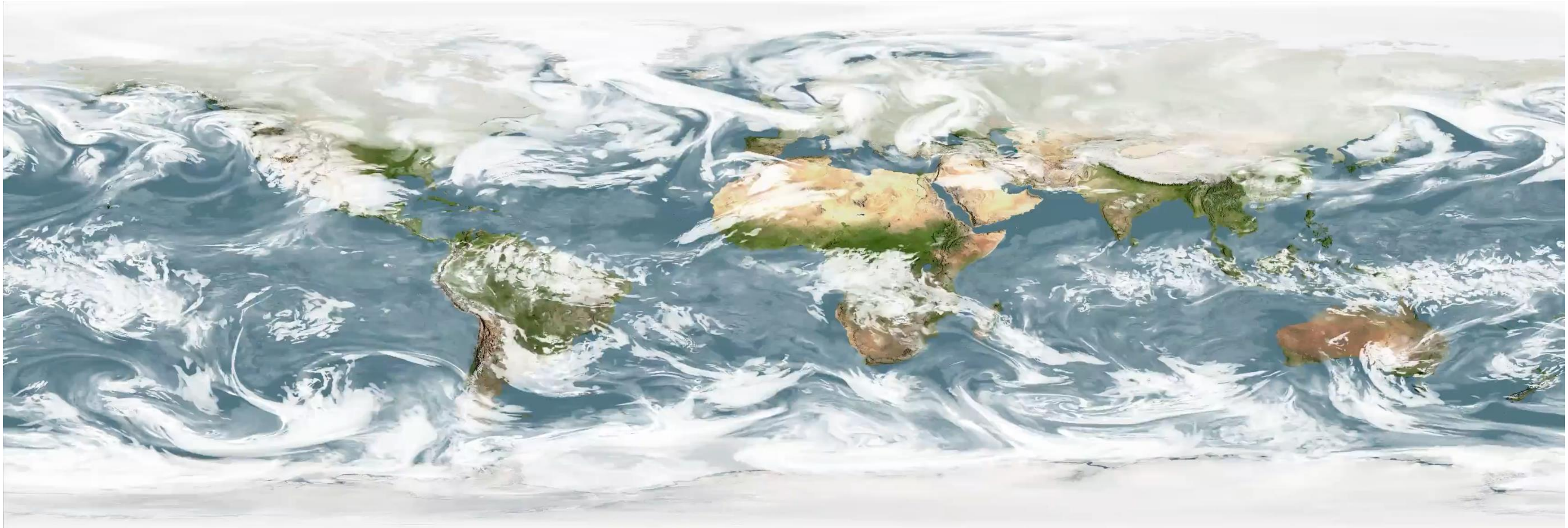
where $\hat{\boldsymbol{k}} = \hat{k}\mathbf{i} + \hat{l}\mathbf{j}$ and $\widehat{\omega}$ is the non-dimensional frequency, and substitute into (3.100), giving

$$\begin{pmatrix} -\mathrm{i}\widehat{\omega} & -\hat{f}_0 & \mathrm{i}\hat{c}^2\hat{k} \\ \hat{f}_0 & -\mathrm{i}\widehat{\omega} & \mathrm{i}\hat{c}^2\hat{l} \\ \mathrm{i}\hat{k} & \mathrm{i}\hat{l} & -\mathrm{i}\widehat{\omega} \end{pmatrix} \begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{\eta} \end{pmatrix} = 0. \tag{3.102}$$

Shallow Water Waves on f-plane

$$\omega^2 = f_0^2 + gH(k^2 + l^2) \quad .$$





GEOS model

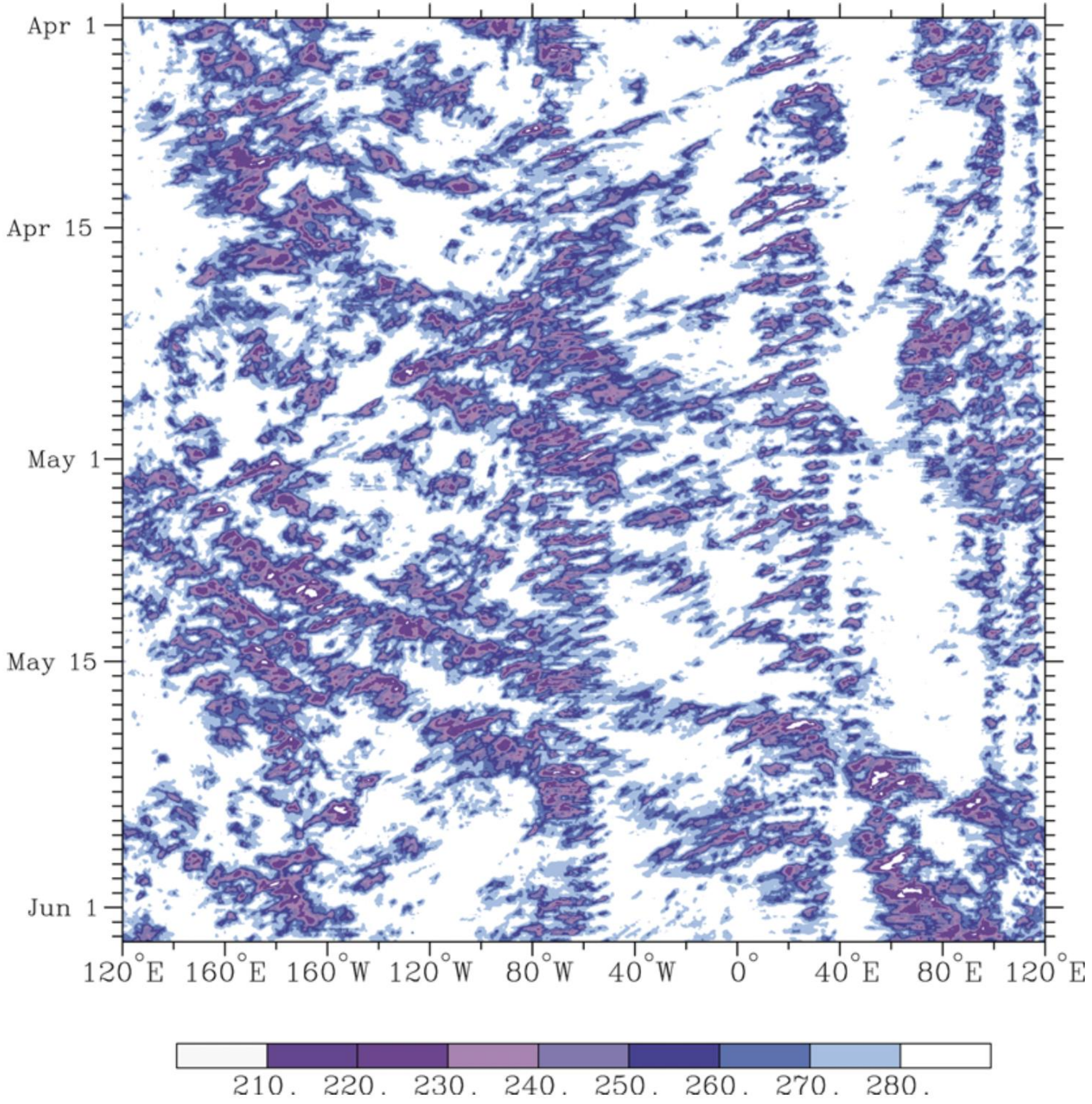


Figure 6. Time-longitude section of CLAUS brightness temperature T_b, averaged from 2.5°S to 7.5°N, from 1 April through 2 June 1987. T_b shading scale is shown at the bottom in °K.

Kiladis (2009)

Shallow Water Waves on β-plane

Quasi-Geostrophic Motions in the Equatorial Area*

By Taroh Matsuno

Geophysical Institute, Tokyo University, Tokyo

(*Manuscript received 15 November 1965, in revised form 11 January 1966*)

$$\frac{\partial u}{\partial t} - \beta y v + \frac{\partial \phi}{\partial x} = 0$$

$$\frac{\partial v}{\partial t} + \beta y u + \frac{\partial \phi}{\partial y} = 0$$

$$\frac{\partial \phi}{\partial t} + c^2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$

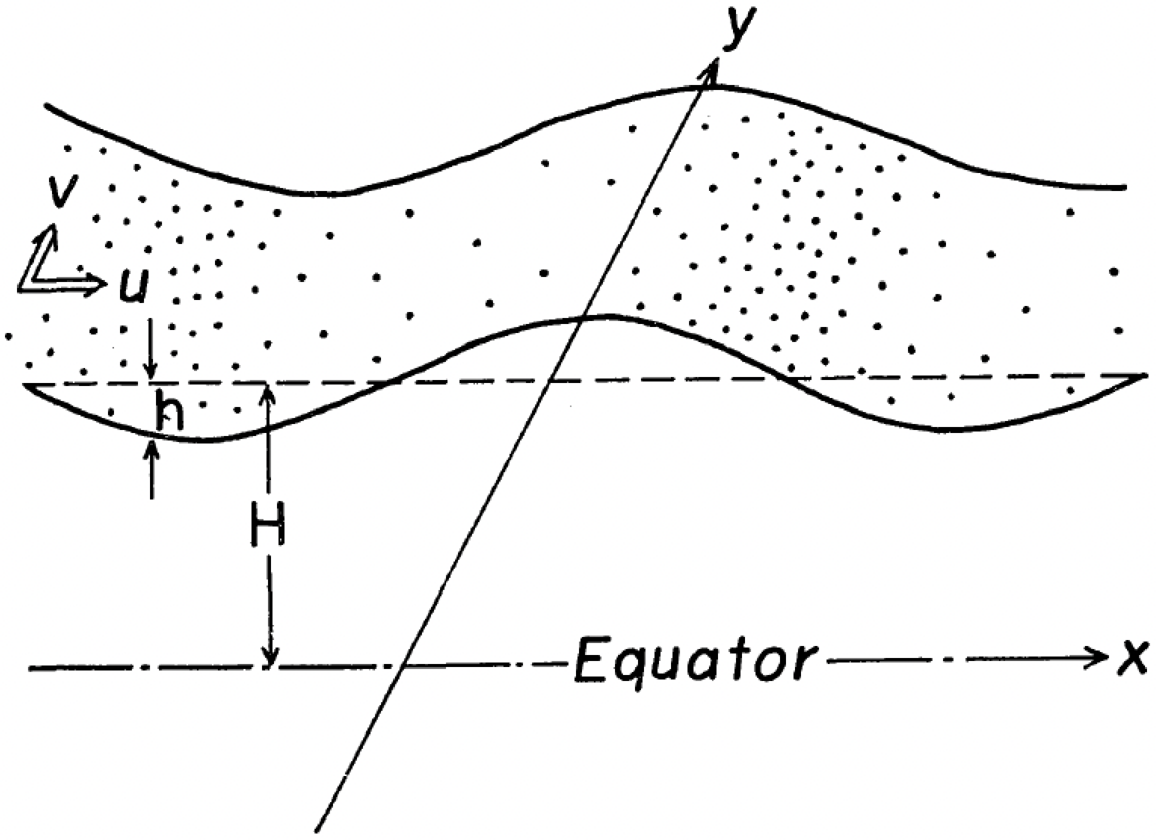
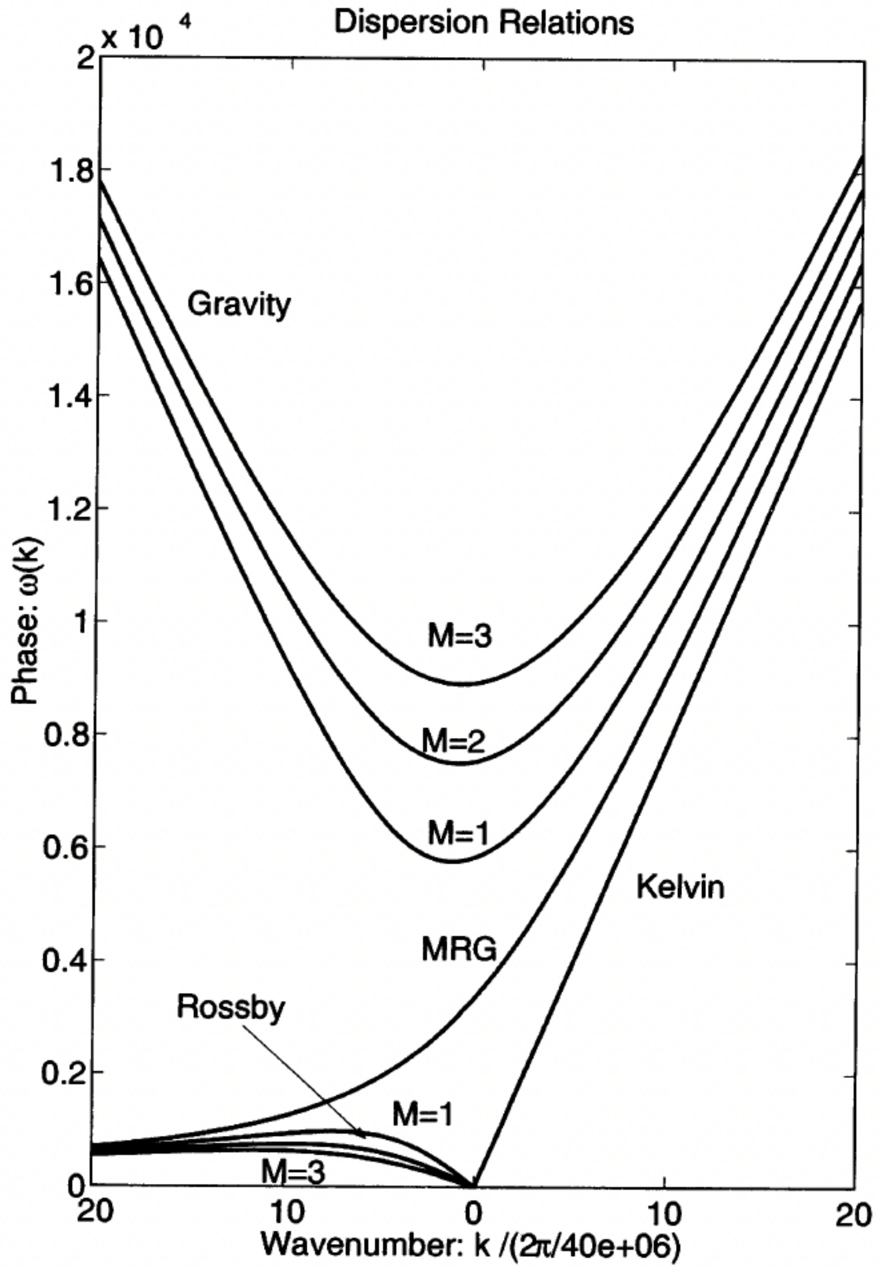
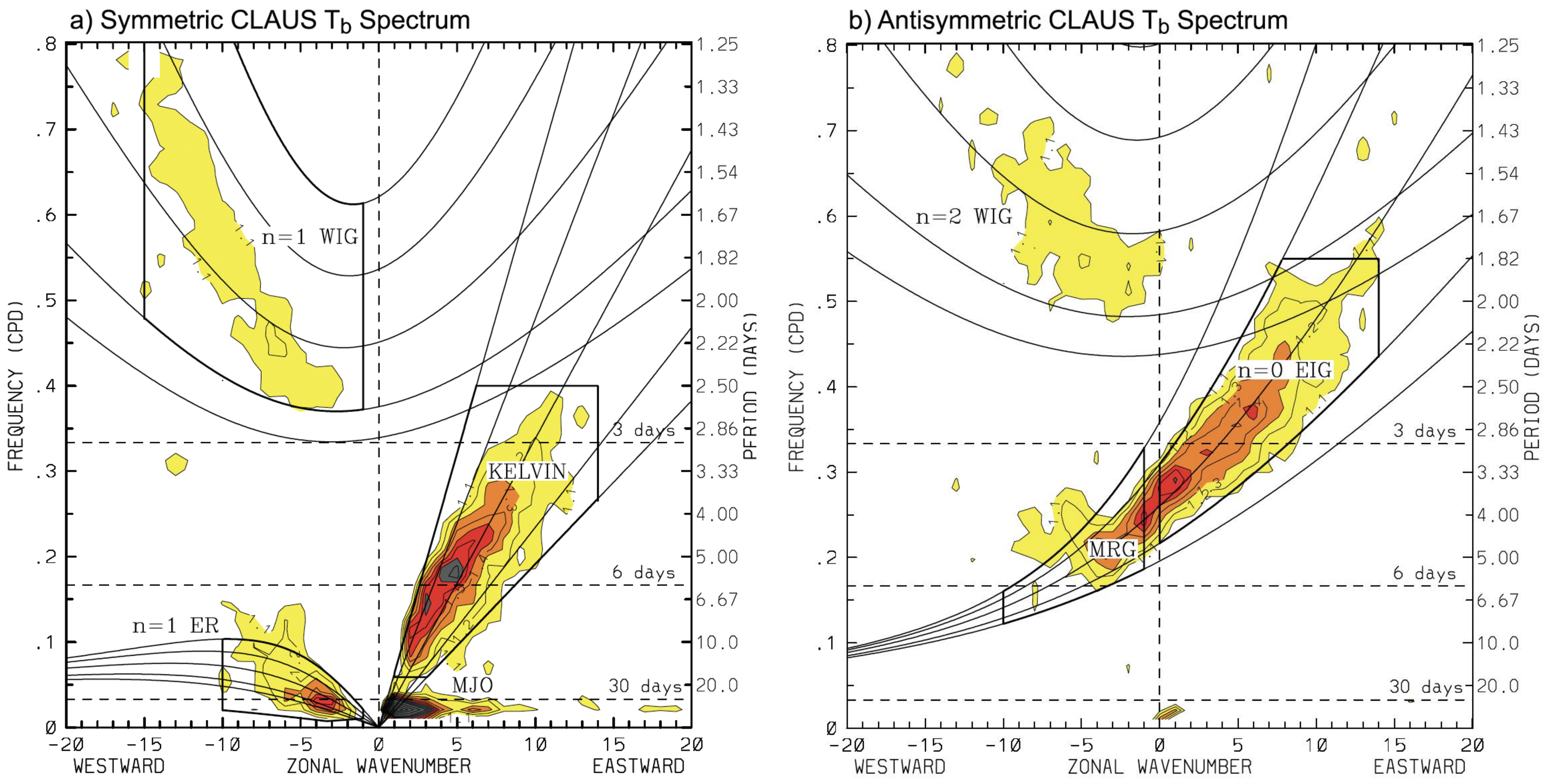


Fig. 1. Model and Coordinates.

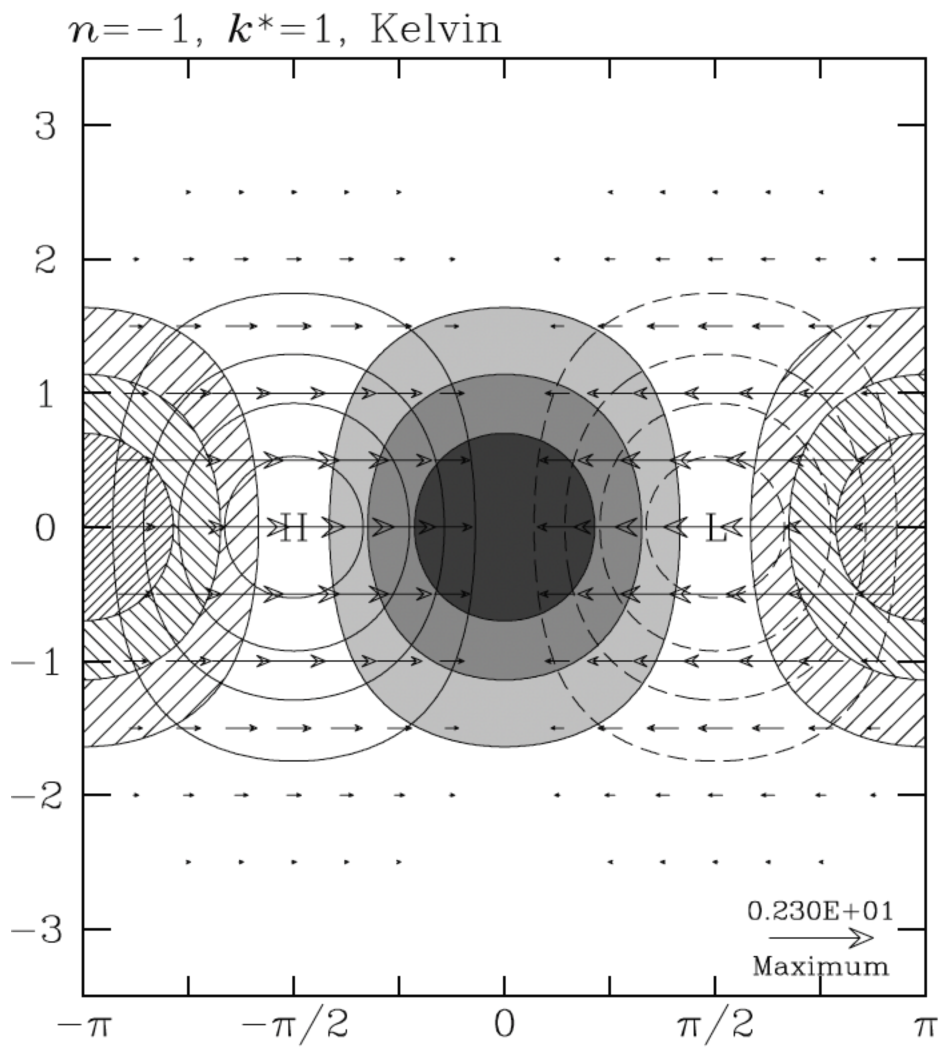


Dispersion Relations
x 10 4
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Phase: ω(k)
Gravity
M=3
M=2
M=1
Kelvin
MRG
Rossby
M=1
M=3
20
10
0
10
20
Wavenumber: k /(2π/40e+06)

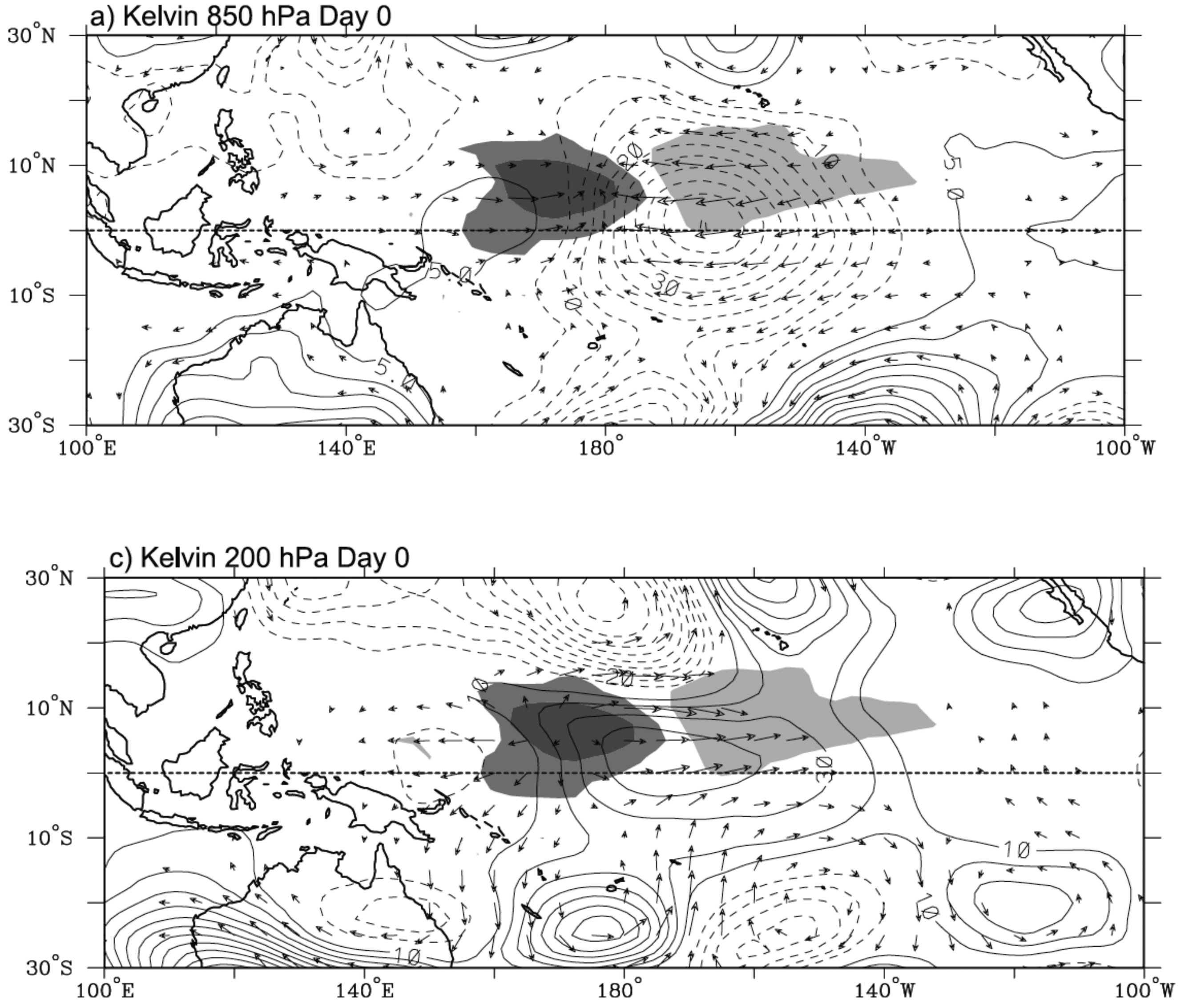


a) Symmetric CLAUS T_b Spectrum
b) Antisymmetric CLAUS T_b Spectrum
n=1 WIG
KELVIN
n=1 ER
MJO
n=2 WIG
n=0 EIG
MRG
3 days
6 days
30 days
FREQUENCY (CPD)
PERIOD (DAYS)
WESTWARD
ZONAL WAVENUMBER
EASTWARD

Kelvin Wave

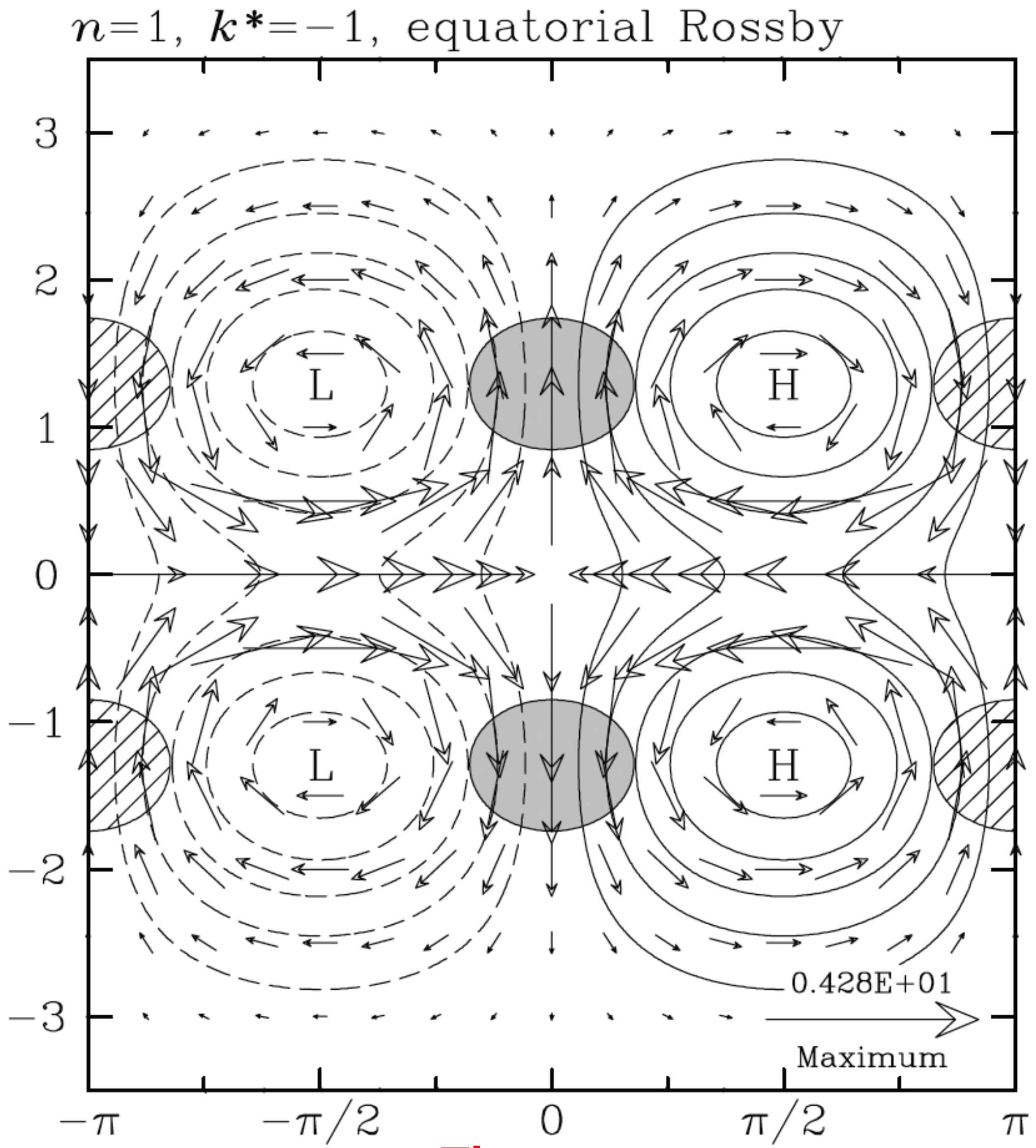


Theory

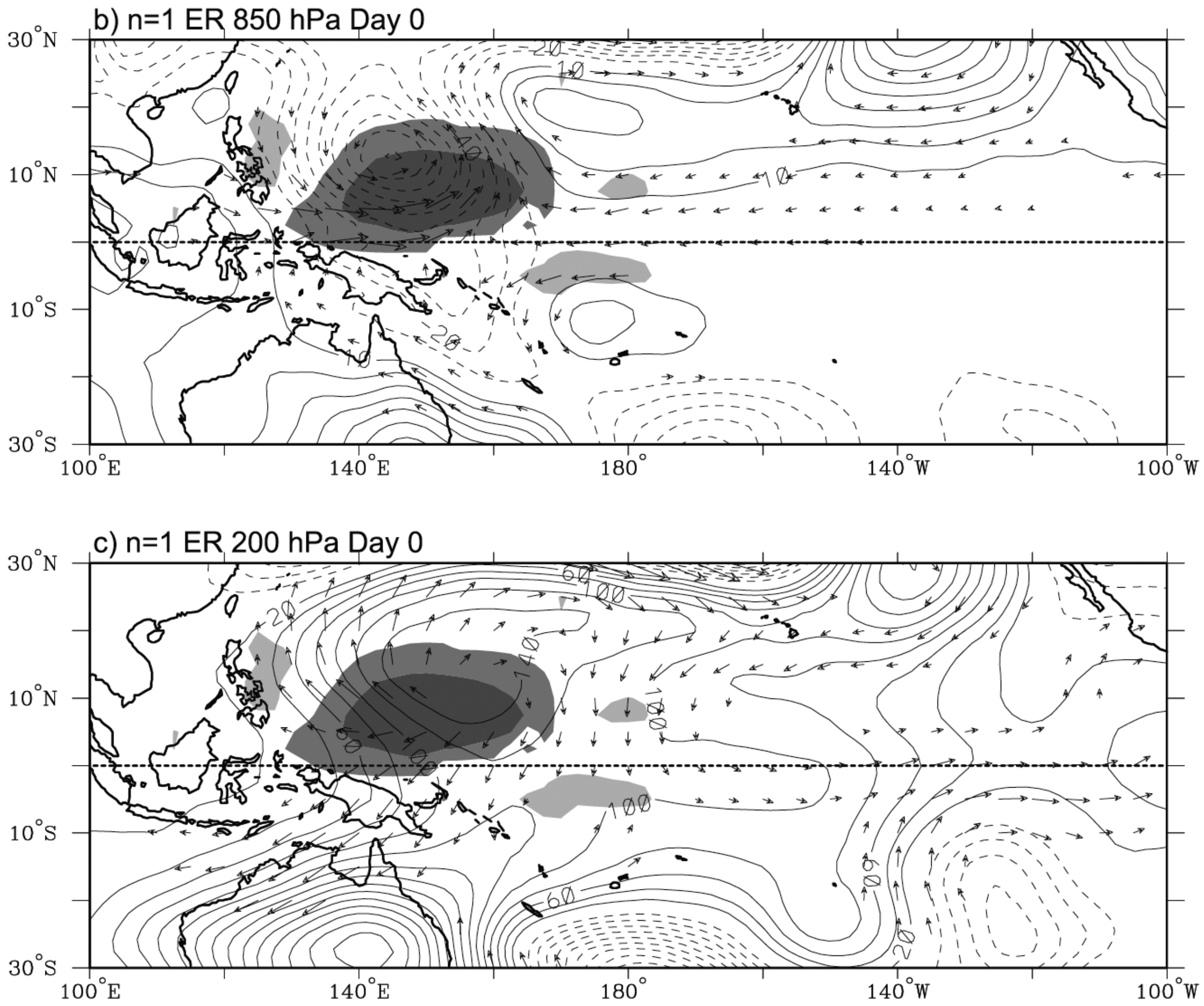


OBS

Rossby Wave

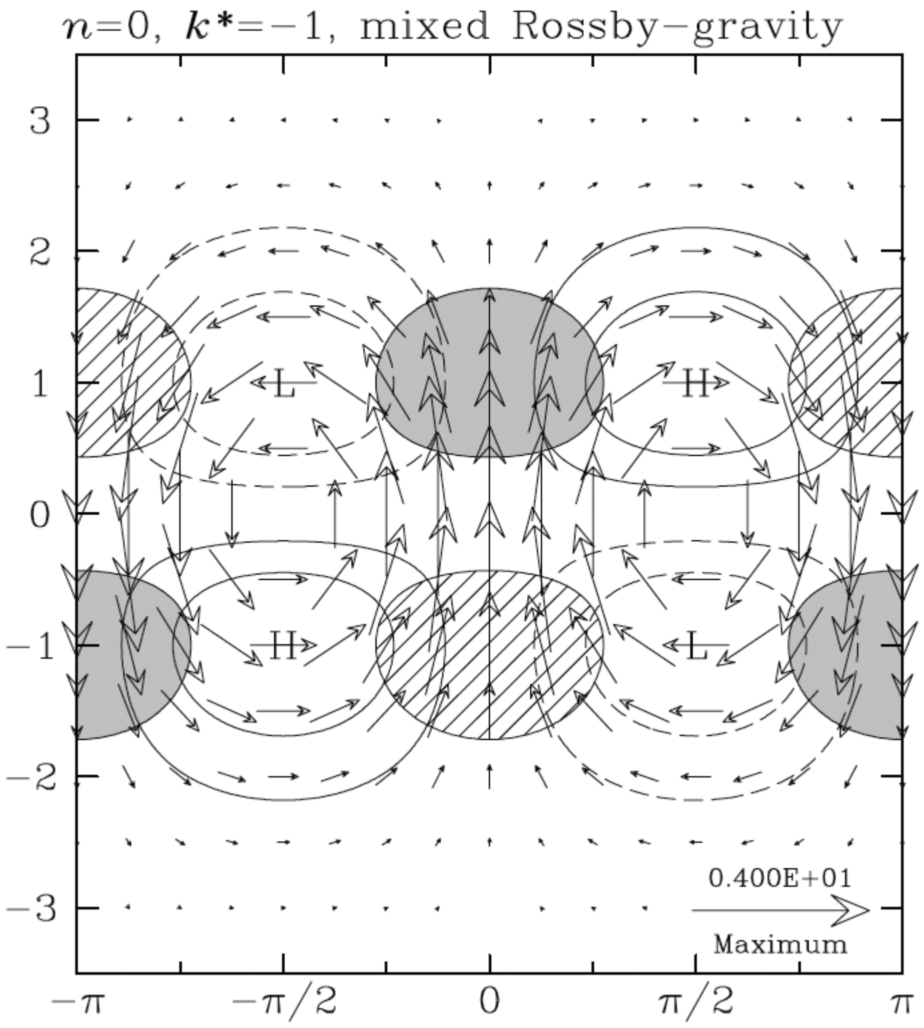


Theory

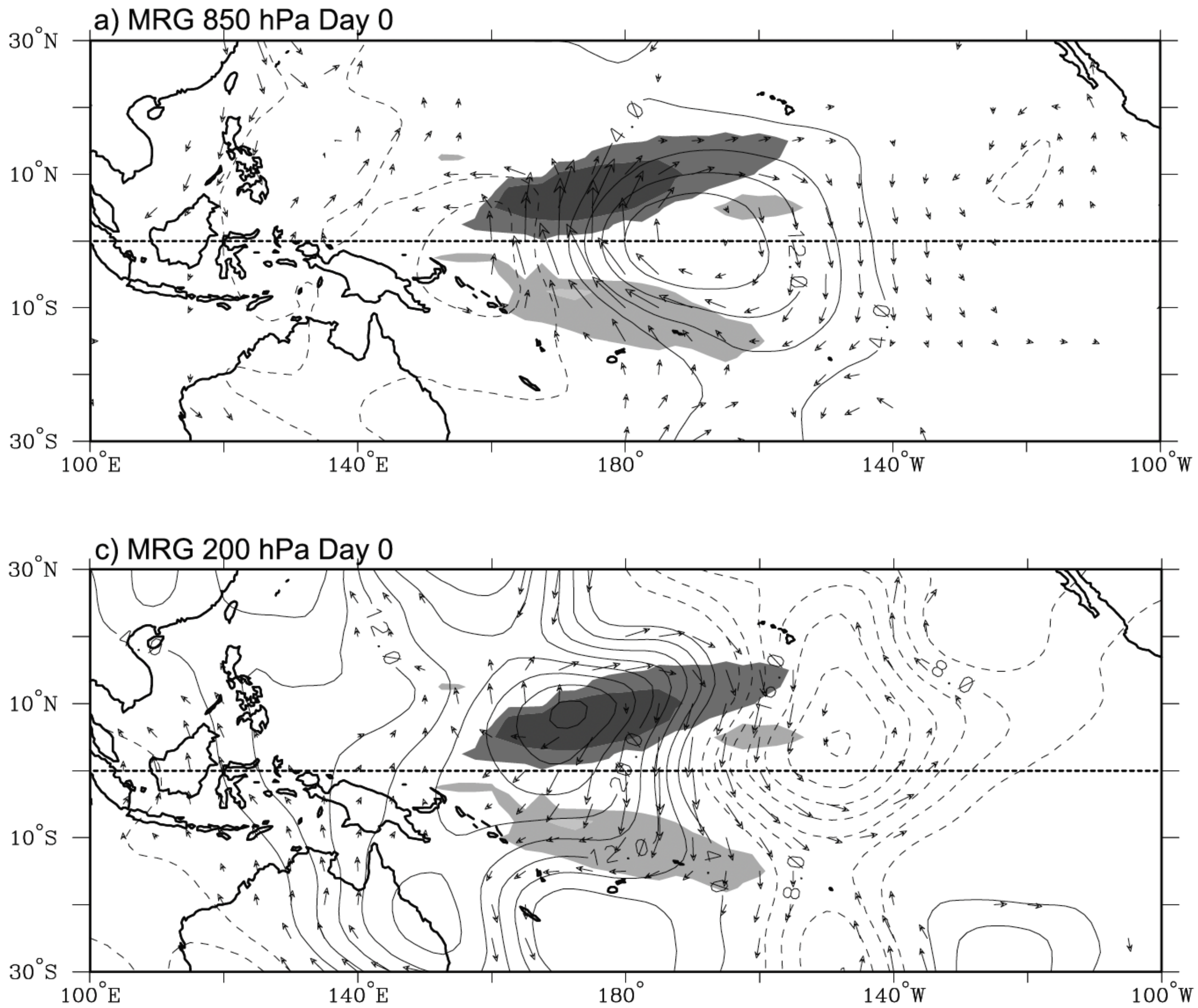


OBS

Mixed Rossby-Gravity Wave

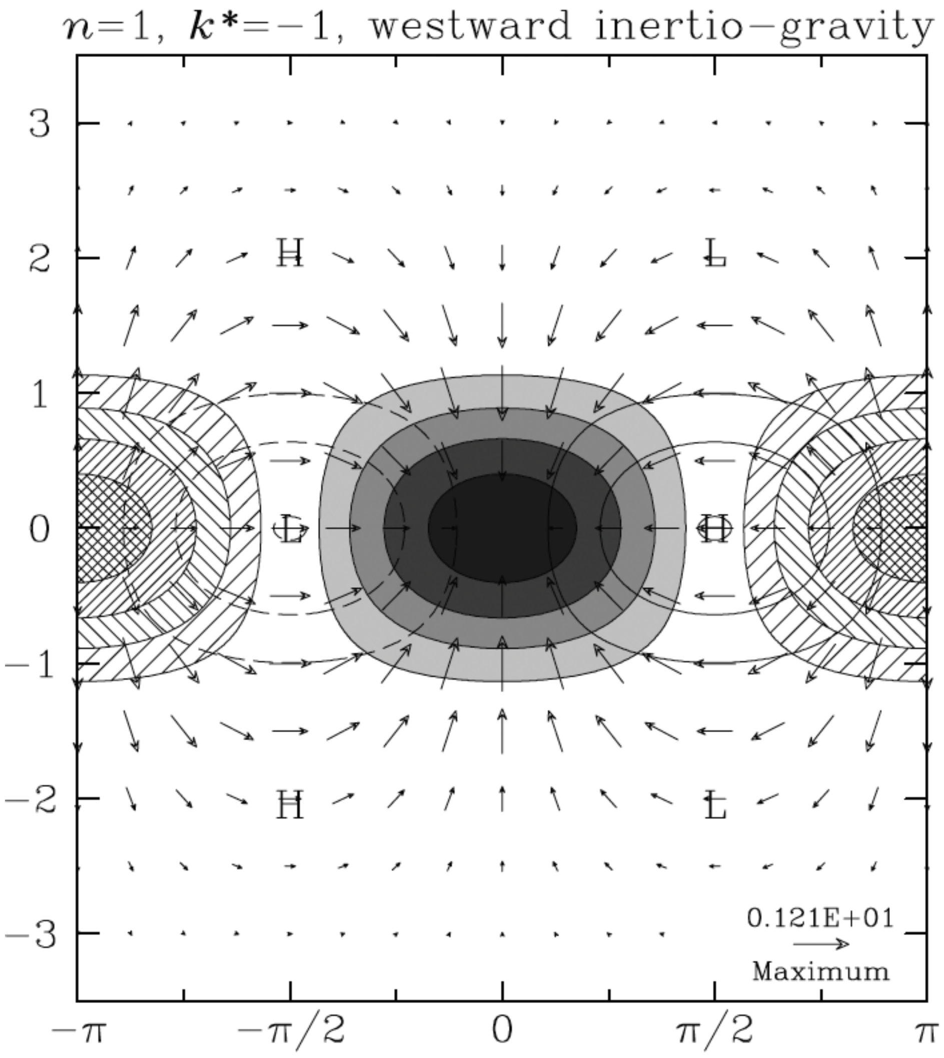


Theory

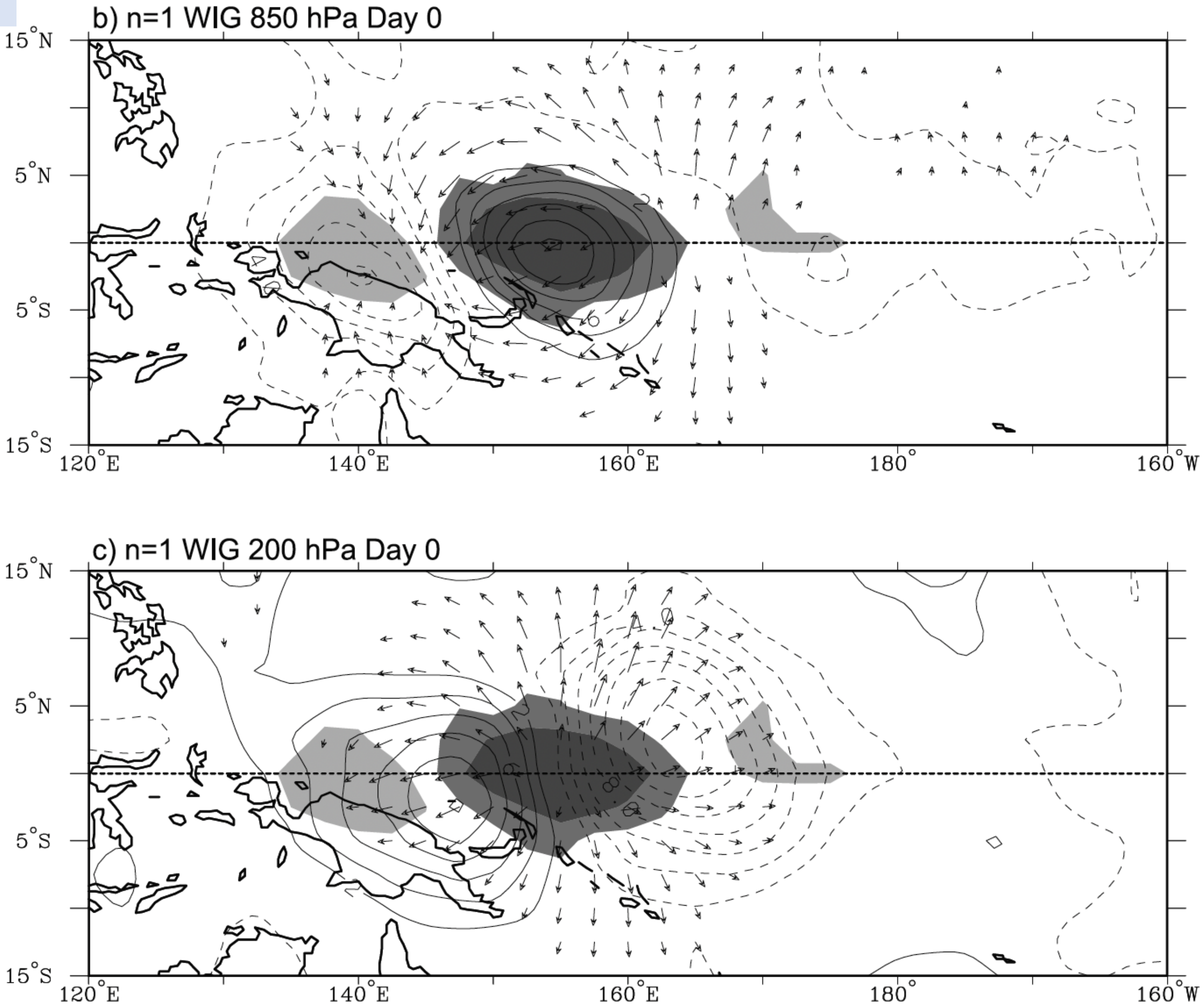


OBS

Westward Inertio-Gravity Wave

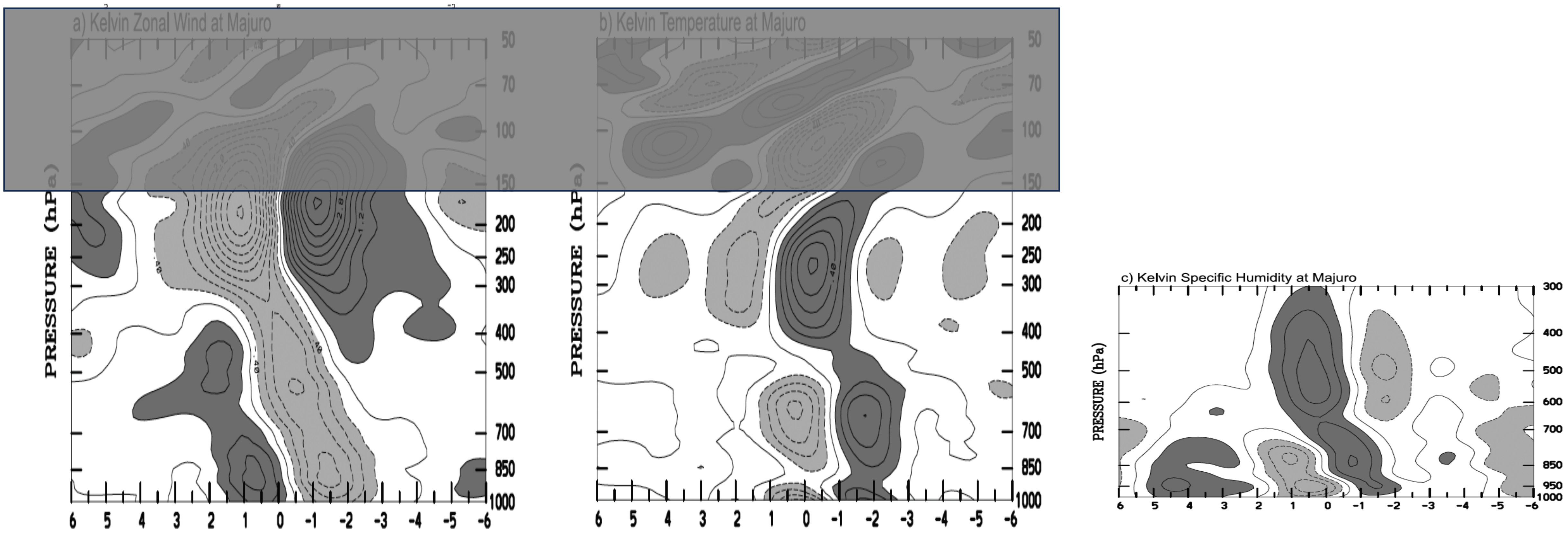


Theory



OBS

Tilted Vertical Structure



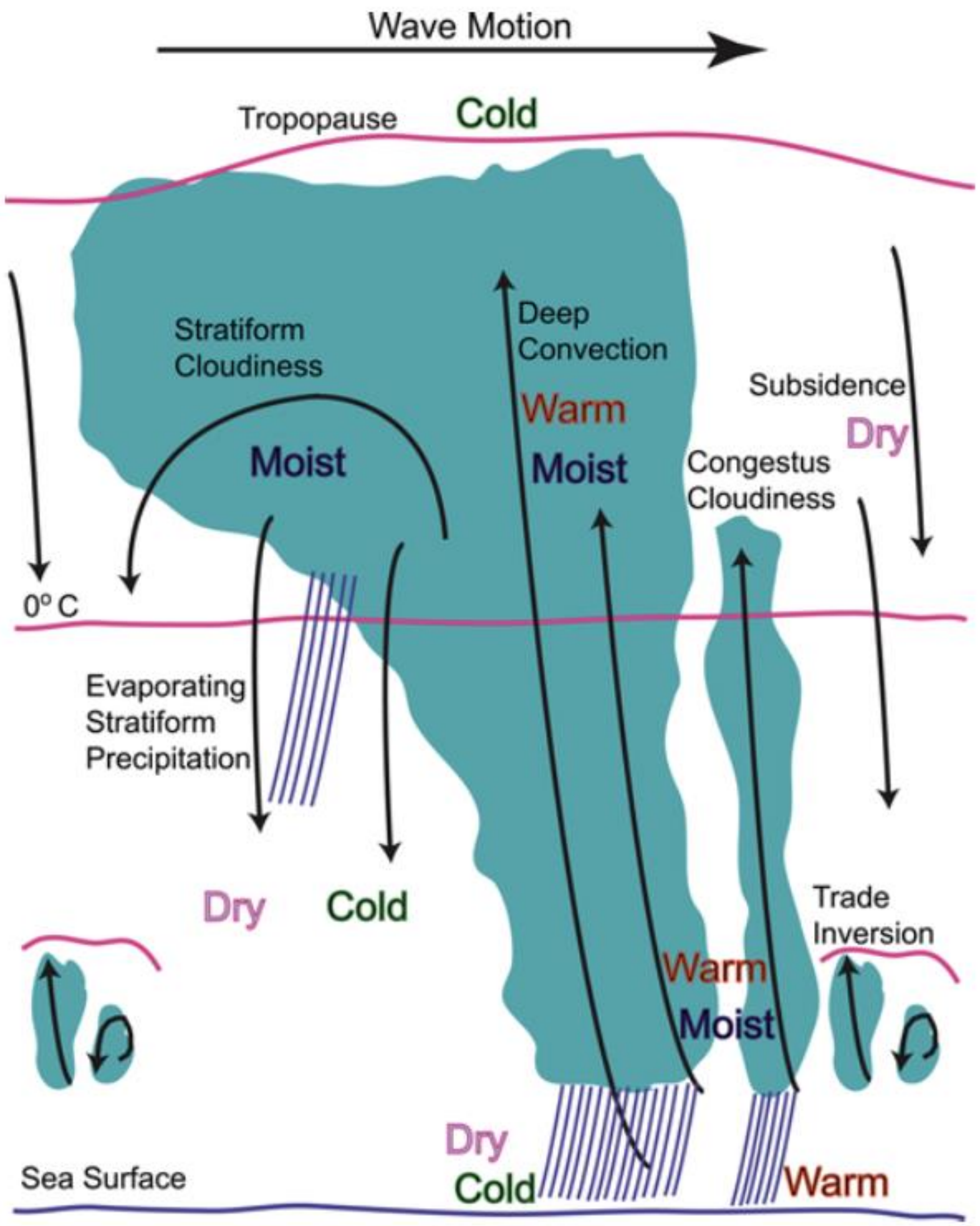
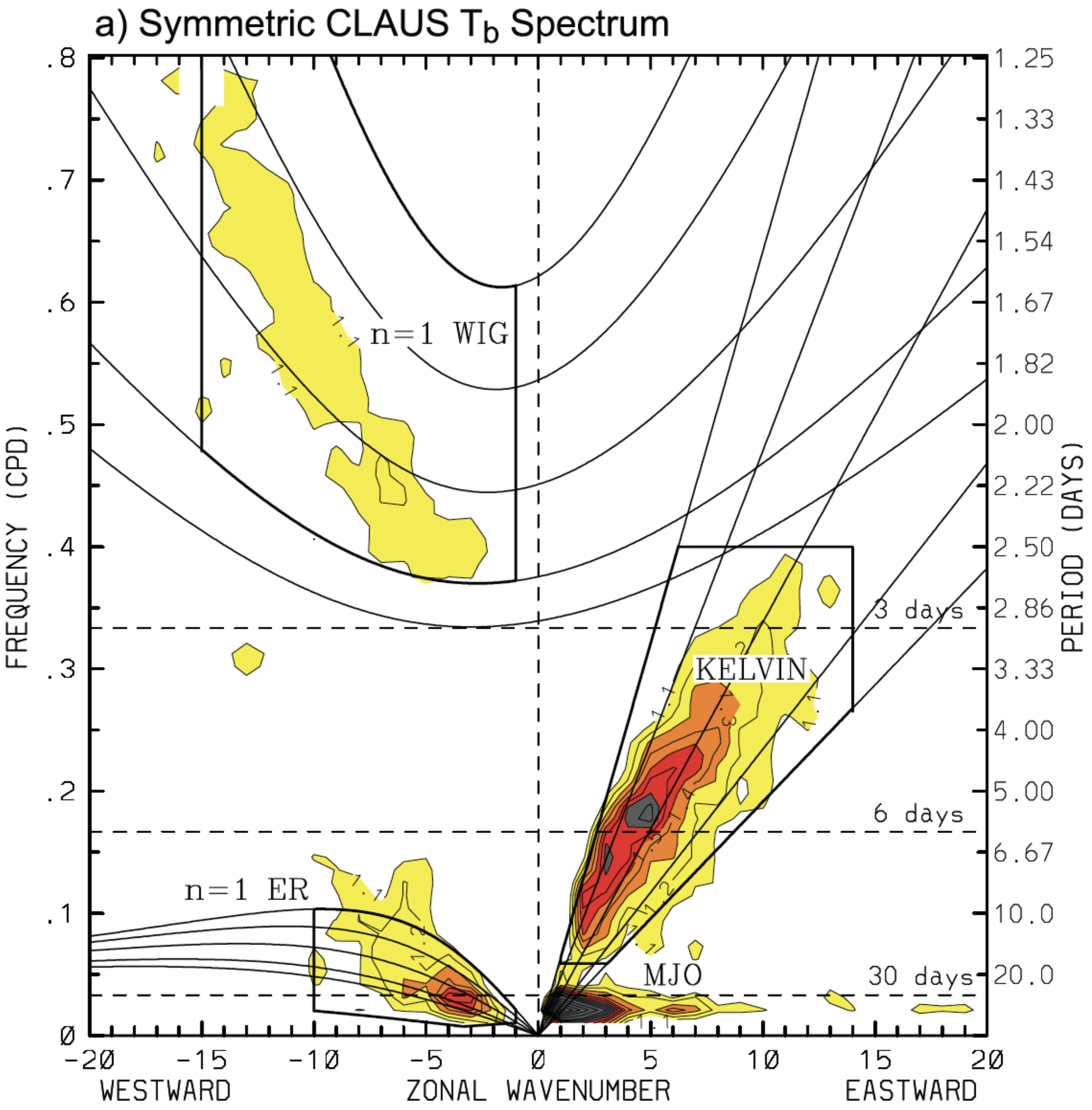


Figure 19. The hierarchy of cloudiness, temperature, and humidity within CCEWs, valid from MCS to MJO scales. Wave movement is from left to right (adapted from *Johnson et al.* [1999], *Straub and Kiladis* [2003c], and *Khouider and Majda* [2008]).



Scale Selection?

MJO?

Equatorial Convectively Coupled Waves in a Simple Multicloud Model

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(Manuscript received 31 January 2008, in final form 14 April 2008)

Turbulent drag
Rayleigh damping
Stratiform heating
Deep convective heating
Radiative cooling
Congestus heating
Precipitation
Downdrafts
Evaporation

$$\frac{\partial \mathbf{v}_1}{\partial t} + \beta y \mathbf{v}_1^{\perp} - \nabla \theta_1 = -C_d u_0 \mathbf{v}_1 - \frac{1}{\tau_R} \mathbf{v}_1$$

$$\frac{\partial \theta_1}{\partial t} - \nabla \cdot \mathbf{v}_1 = H_d + \xi_s H_s + \xi_c H_c + S_1$$

$$\frac{\partial \mathbf{v}_2}{\partial t} = \beta y \mathbf{v}_2^{\perp} - \nabla \theta_2 = -C_d u_0 \mathbf{v}_2 - \frac{1}{\tau_R} \mathbf{v}_2$$

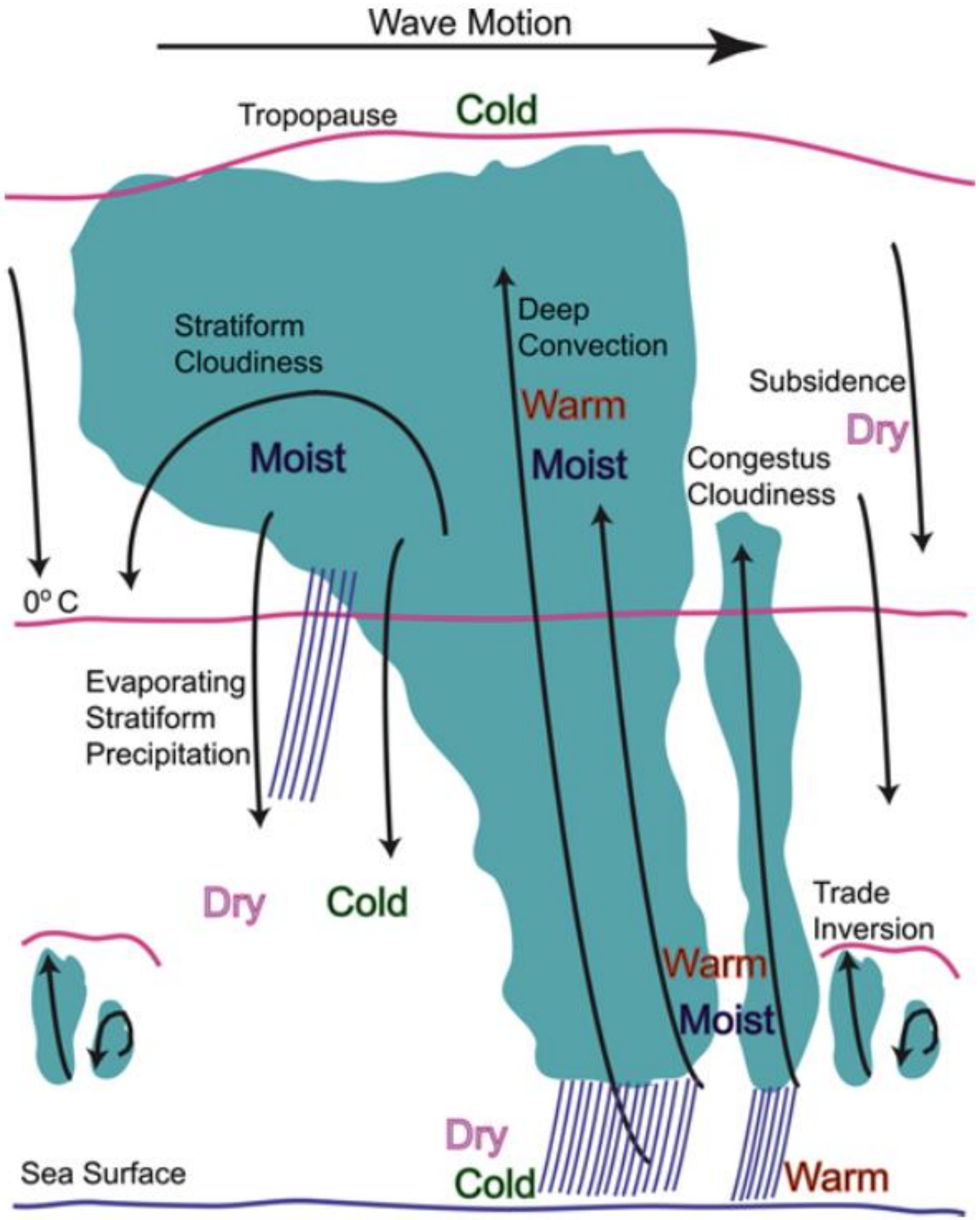
$$\frac{\partial \theta_2}{\partial t} - \frac{1}{4} \nabla \cdot \mathbf{v}_2 = (-H_s + H_c) + S_2$$

$$\frac{\partial q}{\partial t} + \nabla \cdot [(\mathbf{v}_1 + \tilde{\alpha} \mathbf{v}_2) q + \tilde{Q}(\mathbf{v}_1 + \tilde{\lambda} \mathbf{v}_2)] = -P + \frac{D}{H_T}; \quad P = \frac{2\sqrt{2}}{\pi}(H_d + \xi_s H_s + \xi_c H_c)$$

$$\frac{\partial \theta_{eb}}{\partial t} = \frac{1}{h_b}(E - D).$$

$$\mathbf{V}(x, y, z, t) = \sqrt{2}\mathbf{v}_1(x, y, t) \cos z + \sqrt{2}\mathbf{v}_2(x, y, t) \cos 2z \quad \text{and}$$

$$\Theta(x, y, t, z) = z + \sqrt{2}\theta_1(x, y, t) \sin z + 2\sqrt{2}\theta_2(x, y, t) \sin 2z, \quad 0 \leq z \leq \pi.$$



$$\partial_t H_c = \tau_c^{-1}(\alpha_c \Lambda Q_c^+ - H_c)$$

$$H_d = (1 - \Lambda) Q_d^+$$

$$\partial_t H_s = \tau_s^{-1}(\alpha_s H_d - H_s)$$

Λ =1 if $\theta_{eb} - \theta_{em} \geq 20$ K
=0 if $\theta_{eb} - \theta_{em} \leq 10$ K
Linear continuous for 10 K $\leq \theta_{eb} - \theta_{em} \leq 20$ K

Linear Instability Analysis

$$\frac{\partial \mathbf{v}_1}{\partial t} + \beta y \mathbf{v}_1^{\perp} - \nabla\theta_1 = -C_d u_0 \mathbf{v}_1 - \frac{1}{\tau_R}\mathbf{v}_1$$

$$\frac{\partial \theta_1}{\partial t} - \nabla\cdot\mathbf{v}_1 = H_d + \xi_s H_s + \xi_c H_c + S_1$$

$$\frac{\partial \mathbf{v}_2}{\partial t} = \beta y \mathbf{v}_2^{\perp} - \nabla\theta_2 = -C_d u_0 \mathbf{v}_2 - \frac{1}{\tau_R}\mathbf{v}_2$$

$$\frac{\partial \theta_2}{\partial t} - \frac{1}{4}\nabla\cdot\mathbf{v}_2 = (-H_s + H_c) + S_2$$

$$\frac{\partial q}{\partial t} + \nabla\cdot[(\mathbf{v}_1 + \tilde{\alpha}\mathbf{v}_2)q + \tilde{Q}(\mathbf{v}_1 + \tilde{\lambda}\mathbf{v}_2)] = -P + \frac{D}{H_T}; \quad P = \frac{2\sqrt{2}}{\pi}(H_d + \xi_s H_s + \xi_c H_c)$$

$$\frac{\partial \theta_{eb}}{\partial t} = \frac{1}{h_b}(E - D).$$

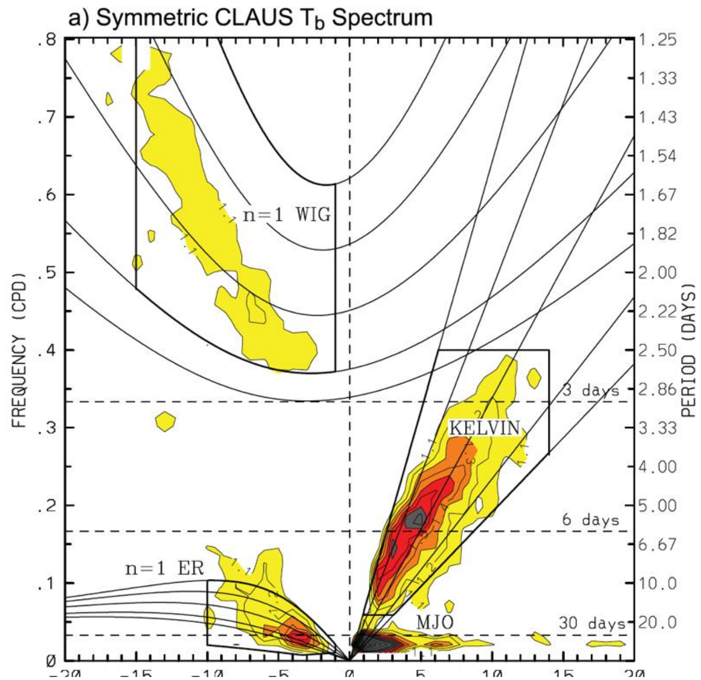
$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{A}\frac{\partial \mathbf{U}}{\partial x} + \mathbf{R}_1\frac{\partial \mathbf{U}}{\partial y} + \mathbf{R}_2(y\mathbf{U}) = \mathbf{B}\mathbf{U}, \qquad (3.3)$$

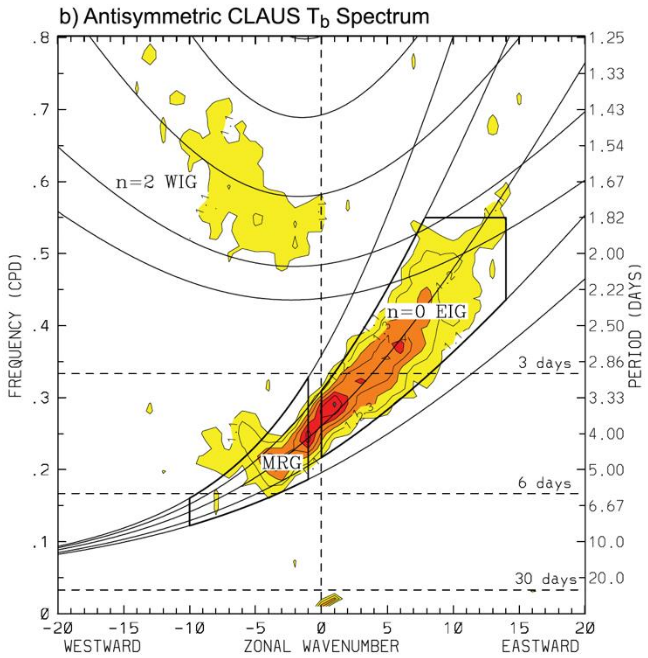
where $\mathbf{U} = (u_1, v_1, u_2, \theta_1, \theta_2, \theta_{eb}, q, H_s, H_c)$ represents a perturbation about the RCE solution and $\mathbf{A}, \mathbf{R}_1, \mathbf{R}_2, \mathbf{B}$ are 10×10 matrices representing the different forcing terms in the multicloud equations. To solve for linear

2004). Next, we seek plane wave solutions of the form

$$\mathcal{U}(x,t) = \mathcal{U}\exp[I(kx - \omega t)], \quad I^2 = -1,$$

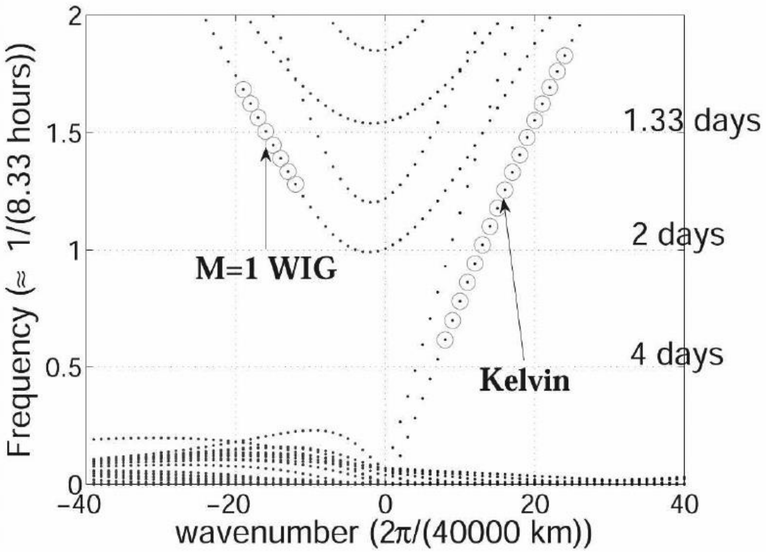
where k is the zonal wavenumber, with $k = 1$ corresponding to a wavelength spanning the equatorial ring, and ω is the generalized phase such that real$(\omega)/k$ is the phase speed and imag(ω) is the growth rate.

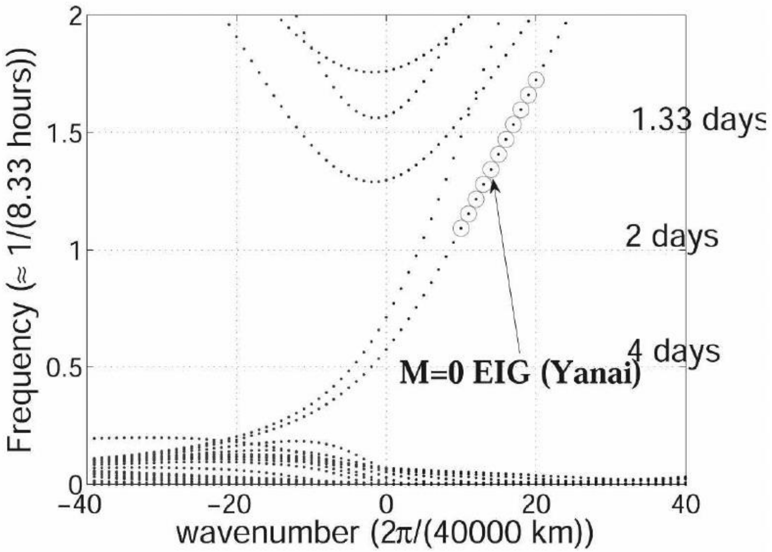




Symmetric

Anti-symmetric

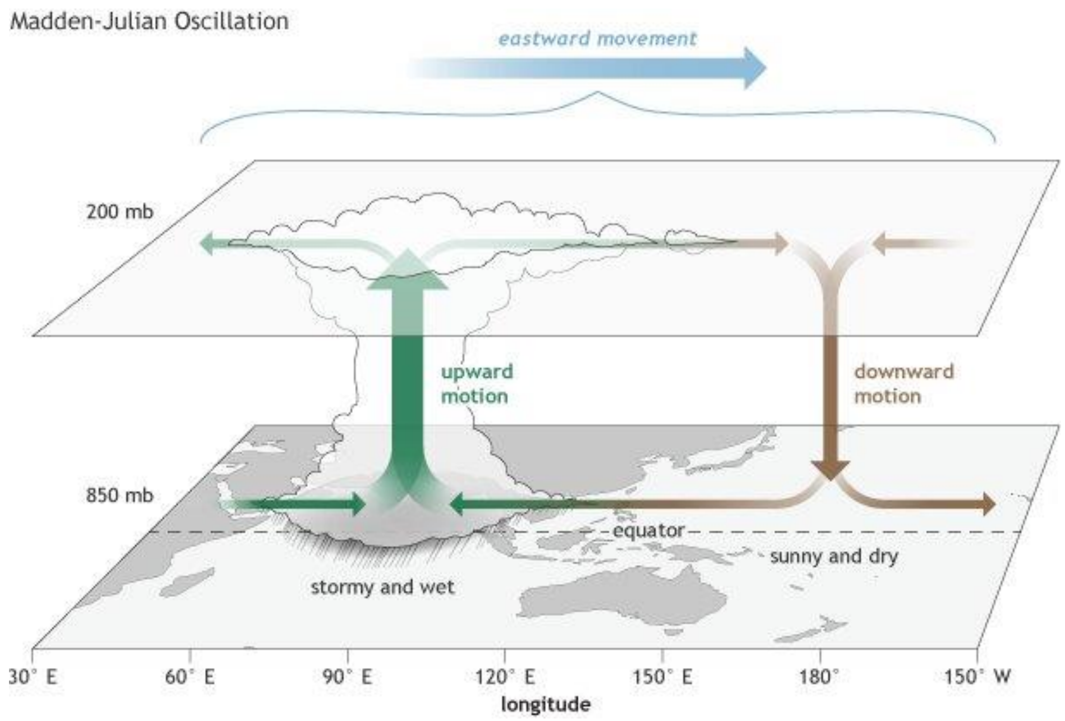




Four Theories of the Madden-Julian Oscillation

C. Zhang[1], Á. F. Adames[2], B. Khouider[3], B. Wang[4], and D. Yang[5,6]

[1]NOAA Pacific Marine Environmental Laboratory, Seattle, WA, USA, [2]Department of Climate and Space Sciences and Engineering, University of Michigan, Ann Arbor, MI, USA, [3]Department of Mathematics and Statistics, University of Victoria, Victoria, British Columbia, Canada, [4]Department of Atmospheric Sciences, University of Hawaii, Honolulu, HI, USA, [5]Department of Land, Air and Water Resources, University of California, Davis, CA, USA, [6]Lawrence Berkeley National Laboratory, Berkeley, CA, USA



Common Framework of MJO Theories

$$\frac{\partial u}{\partial t} - \beta y v = -\frac{\partial \phi}{\partial x} - \epsilon u$$

$$\frac{\partial v}{\partial t} + \beta y u = -\frac{\partial \phi}{\partial y} - \epsilon v$$

$$\frac{\partial \phi}{\partial t} + H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = Q_1 - \mu\phi$$

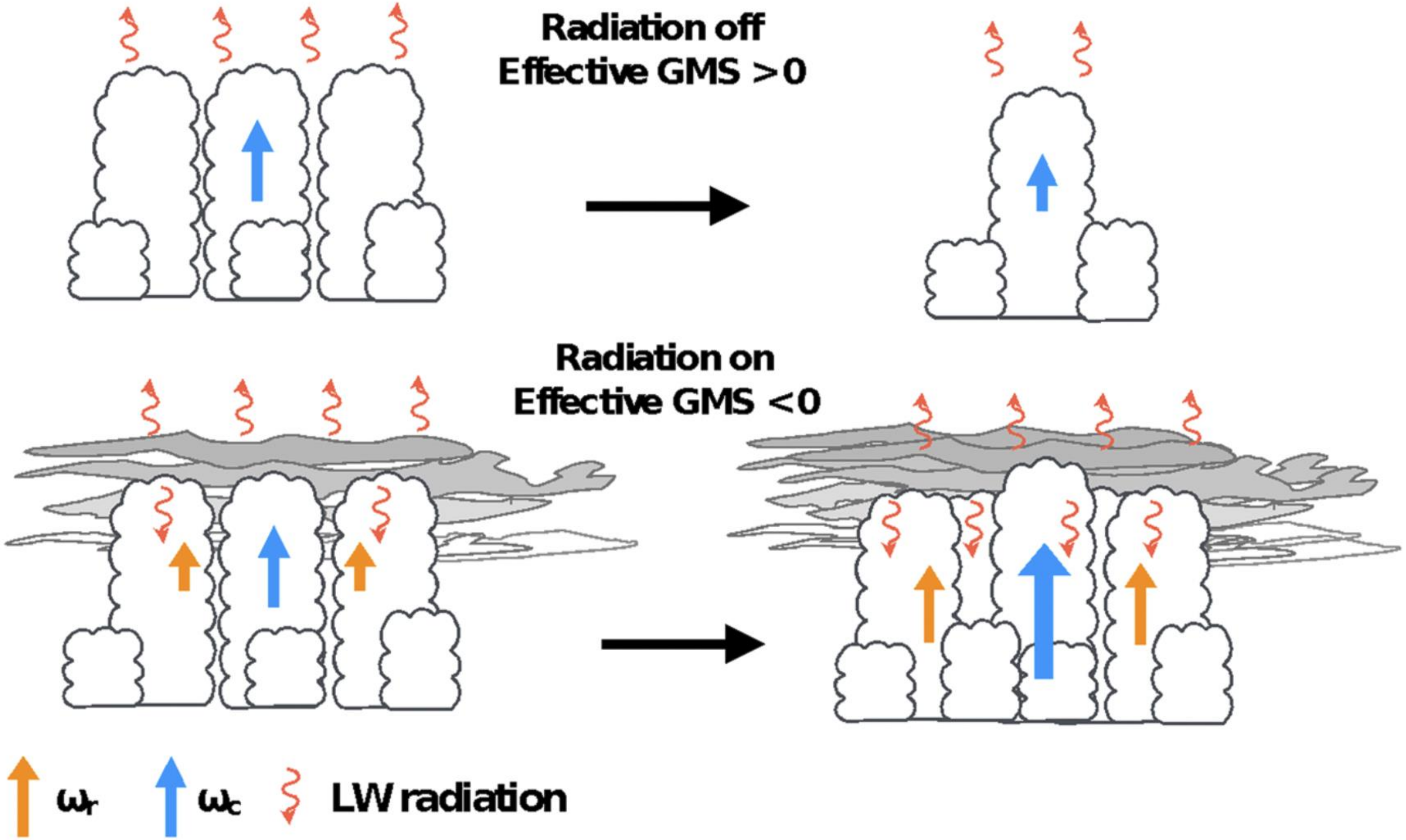
$$\frac{Dq}{Dt} = -Q_2$$

Table 1
Main Assumed Processes and Approximations

	Skeleton	Moisture Mode	Gravity Wave	Trio Interaction
Internal and first baroclinic mode	√	√	√	√
Equatorial β-plane	√	√	√	√
Linear dynamics	√	√	√	√
Hydrostatic balance	√	√	√	√
Resting basic state	√		√	√
Wave activity tendency	√			
Cloud-radiative heating		√		√
Horizontal moisture advection		√		√
Longwave approximation	√	√		√
Boundary layer dynamics		√		√
No zonal momentum tendency		√		
Weak temperature gradient approximation		√		
Moisture tendency	√	√		√
Prescribed Rossby-Kelvin structure	√	√		
Positive only precipitation anomalies			√	√
Linear damping of momentum		√		√
Newtonian cooling		√		√
Radiative-convective equilibrium	√	√		
Large-scale envelope of convection	√			
Convective trigger			√	
Boussinesq approximation	√			

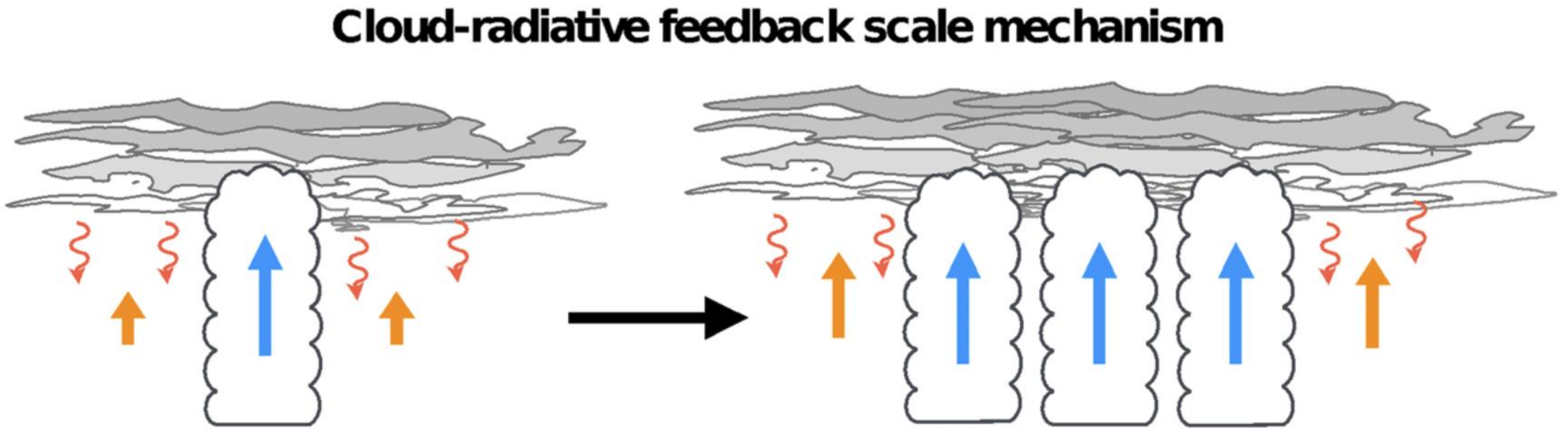
Moist Instability

$$R' = -rP',$$

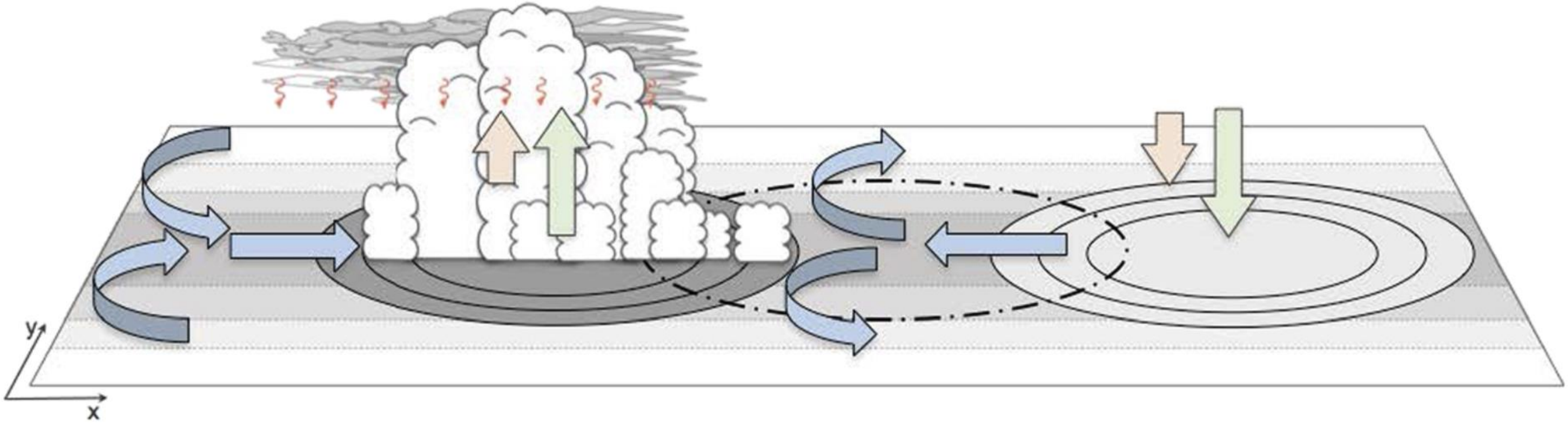


Planetary Scale Selection

$$r \approx r_0 e^{-L_r k},$$



Eastward Propagation

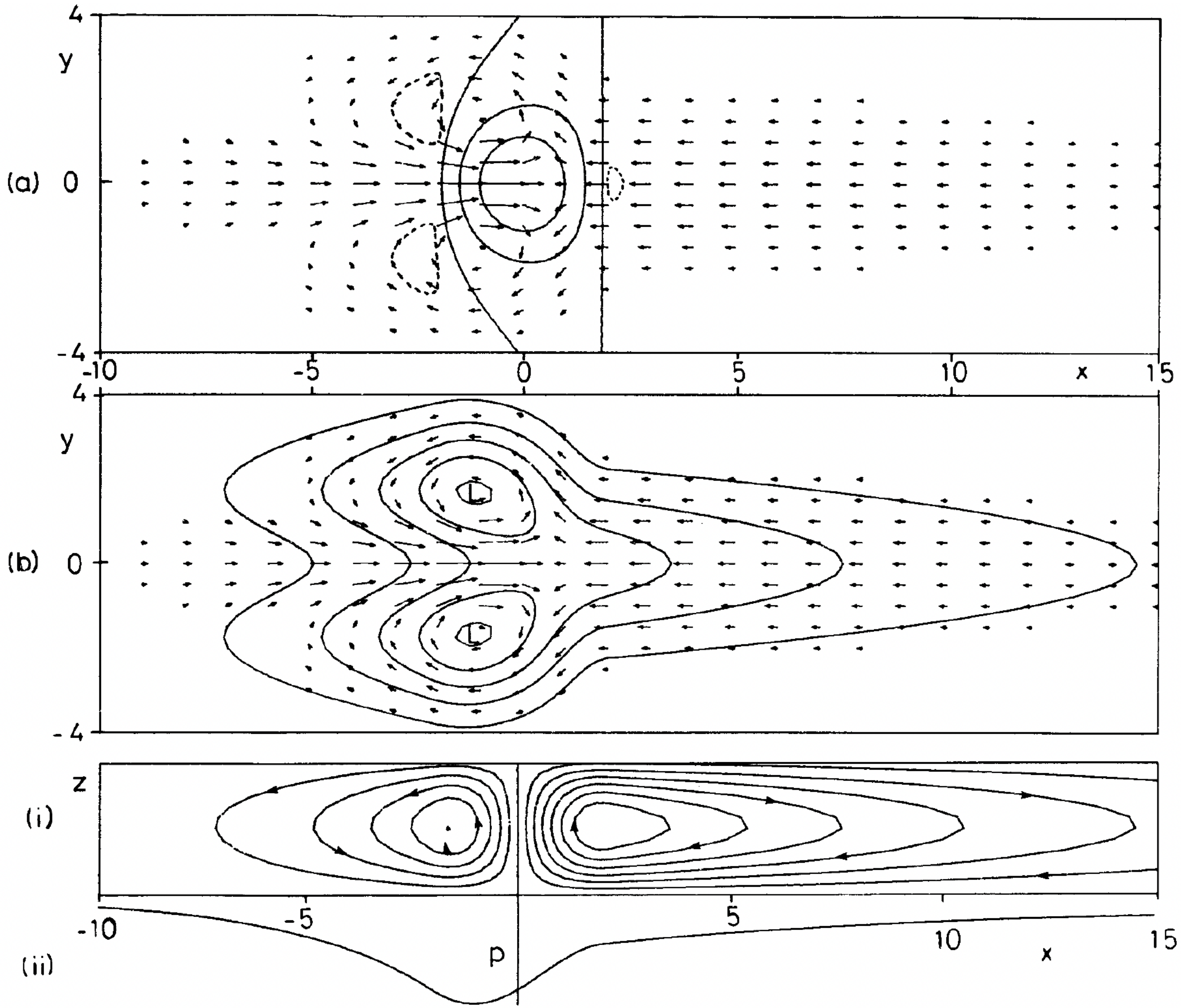


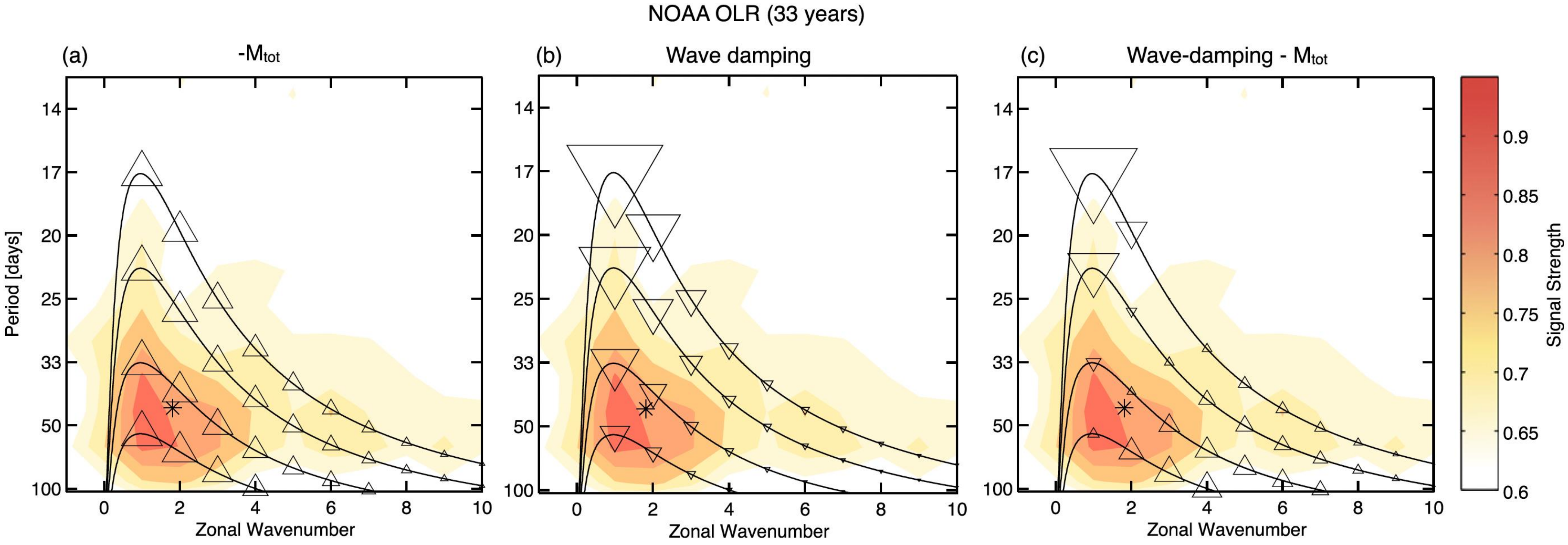
Gill-type Model for MJO winds

$$\epsilon u - \tfrac{1}{2} y v = -\frac{\partial p}{\partial x},$$

$$\epsilon v + \tfrac{1}{2} y u = -\frac{\partial p}{\partial y},$$

$$\epsilon p + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -Q,$$




NOAA OLR (33 years)
(a)
$-M_{tot}$
(b)
Wave damping
(c)
Wave-damping - M_{tot}
Period [days]
14
17
20
25
33
50
100
0
2
4
6
8
10
Zonal Wavenumber
Signal Strength
0.9
0.85
0.8
0.75
0.7
0.65
0.6