### Recurrent Neural Networks

Often data arise as sequences:

- Documents are sequences of words, and their relative positions have meaning.
- Time-series such as weather data or financial indices.
- Recorded speech or music.
- Handwriting, such as doctor's notes.

RNNs build models that take into account this sequential nature of the data, and build a memory of the past.

- The feature for each observation is a *sequence* of vectors  $X = \{X_1, X_2, \ldots, X_L\}.$
- The target Y is often of the usual kind e.g. a single variable such as Sentiment, or a one-hot vector for multiclass.
- However,  $Y$  can also be a sequence, such as the same document in a different language.



## Simple Recurrent Neural Network Architecture



- The hidden layer is a sequence of vectors  $A_{\ell}$ , receiving as input  $X_{\ell}$  as well as  $A_{\ell-1}$ .  $A_{\ell}$  produces an output  $O_{\ell}$ .
- The same weights  $W$ ,  $U$  and  $B$  are used at each step in the sequence  $-$  hence the term recurrent.
- The  $A_{\ell}$  sequence represents an evolving model for the response that is updated as each element  $X_{\ell}$  is processed.



### When to Use Deep Learning

- CNNs have had enormous successes in image classification and modeling, and are starting to be used in medical diagnosis. Examples include digital mammography, ophthalmology, MRI scans, and digital X-rays.
- RNNs have had big wins in speech modeling, language translation, and forecasting.

Should we always use deep learning models?

- Often the big successes occur when the *signal to noise ratio* is high  $-$  e.g. image recognition and language translation. Datasets are large, and overfitting is not a big problem.
- For noisier data, simpler models can often work better.
	- On the NYSE data, the  $AR(5)$  model is much simpler than a RNN, and performed as well.
	- On the IMDB review data, the linear model fit by glmnet did as well as the neural network, and better than the RNN.
- We endorse the *Occam's razor* principal we prefer simpler models if they work as well. More interpretable!





1. Start with a guess  $\theta^0$  for all the parameters in  $\theta$ , and set  $t = 0$ .

- 2. Iterate until the objective  $R(\theta)$  fails to decrease:
	- (a) Find a vector  $\delta$  that reflects a small change in  $\theta$ , such that  $\theta^{t+1} = \theta^t + \delta$  reduces the objective; i.e.  $R(\theta^{t+1}) < R(\theta^t)$ . (b) Set  $t \leftarrow t + 1$ .

#### Gradients and Backpropagation

 $R(\theta) = \sum_{i=1}^{n} R_i(\theta)$  is a sum, so gradient is sum of gradients.

$$
R_i(\theta) = \frac{1}{2}(y_i - f_{\theta}(x_i))^2 = \frac{1}{2}\left(y_i - \beta_0 - \sum_{k=1}^K \beta_k g(w_{k0} + \sum_{j=1}^p w_{kj} x_{ij})\right)^2
$$

For ease of notation, let  $z_{ik} = w_{k0} + \sum_{j=1}^{p} w_{kj} x_{ij}$ .

Backpropagation uses the *chain rule for differentiation*:

$$
\frac{\partial R_i(\theta)}{\partial \beta_k} = \frac{\partial R_i(\theta)}{\partial f_{\theta}(x_i)} \cdot \frac{\partial f_{\theta}(x_i)}{\partial \beta_k} \n= -(y_i - f_{\theta}(x_i)) \cdot g(z_{ik}).\n\frac{\partial R_i(\theta)}{\partial w_{kj}} = \frac{\partial R_i(\theta)}{\partial f_{\theta}(x_i)} \cdot \frac{\partial f_{\theta}(x_i)}{\partial g(z_{ik})} \cdot \frac{\partial g(z_{ik})}{\partial z_{ik}} \cdot \frac{\partial z_{ik}}{\partial w_{kj}} \n= -(y_i - f_{\theta}(x_i)) \cdot \beta_k \cdot g'(z_{ik}) \cdot x_{ij}.
$$

#### 上节课回顾

#### Tricks of the Trade

- Slow learning. Gradient descent is slow, and a small learning rate  $\rho$  slows it even further. With early stopping, this is a form of regularization.
- Stochastic gradient descent. Rather than compute the gradient using all the data, use a small *minibatch* drawn at random at each step. E.g. for MNIST data, with  $n = 60K$ , we use minibatches of 128 observations.
- An *epoch* is a count of iterations and amounts to the number of minibatch updates such that  $n$  samples in total have been processed; i.e.  $60K/128 \approx 469$  for MNIST.
- Regularization. Ridge and lasso regularization can be used to shrink the weights at each layer. Two other popular forms of regularization are *dropout* and *augmentation*, discussed next.



#### Double Descent

- With neural networks, it seems better to have too many hidden units than too few.
- Likewise more hidden layers better than few.
- Running stochastic gradient descent till zero training error ۰ often gives good out-of-sample error.
- Increasing the number of units or layers and again training till zero error sometimes gives even better out-of-sample error.

What happened to overfitting and the usual bias-variance trade-off?

Belkin, Hsu, Ma and Mandal (arXiv 2018) Reconciling Modern Machine Learning and the Bias-Variance Trade-off.



- When  $d \leq 20$ , model is OLS, and we see usual bias-variance trade-off
- When  $d > 20$ , we revert to minimum-norm. As d increases above 20,  $\sum_{i=1}^{d} \hat{\beta}_i^2$  decreases since it is easier to achieve zero error, and hence less wiggly solutions. 66 of 72





**Goal**: 5 machine learning models are used to forecast six climatic factors on a monthly ahead.

**Data**: The climate data for 72 years (1 January 1951–31 December 2022) used in this study include

1. monthly average atmospheric temperature,

- 2. extreme minimum atmospheric temperature,
- 3. extreme maximum atmospheric temperature,

4. precipitation,

- 5. average relative humidity,
	- 6. sunlight hours.



Figure 5. The CNN-LSTM architecture.



Table 3. Comparison analysis between various models for simulated monthly average atmospheric temperature.



Figure 6. Comparison of observed and predicted values of monthly average atmospheric temperature in Jinan from 2015 to 2022. (a) Plot of the prediction results, (b) Scatterplot of the prediction results.



Table 6. Comparison analysis between various models for simulated monthly precipitation.



Figure 9. Comparison of observed and predicted values of monthly precipitation in Jinan from 2015 to 2022. (a) Plot of the prediction results,  $(b)$  Scatterplot of the prediction results.



Table 9. Average time for each model run.

# Python代码演示

#### **IMDB Document Classification**  $\mathbf{r}$

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We now implement models for sentiment classification (Section???) on the IMDB dataset. As mentioned above code block8, we are using a preprocessed version of the IMDB dataset found in the keras package. As keras uses tensorflow, a different tensor and deep learning library, we have converted the data to be suitable for torch. The code used to convert from keras is available in the module ISLP.torch. make imdb. It requires some of the k packages to run. These data use a dictionary of size 10,000.

We have stored three different representations of the review data for this lab:

- load\_tensor(), a sparse tensor version usable by torch;
- load\_sparse(), a sparse matrix version usable by sklearn, since we will compare with a lasso fit;
- load\_sequential(), a padded version of the original sequence representation, limited to the last 500 words of each review.

 $+31$  cells hidden

#### **Recurrent Neural Networks**

In this lab we fit the models illustrated in Section $\sim$ ???.

 $+56$  cells hidden

Click to add a cell.